Imperial College
London

Position and angular resolution studies with ECAL TB prototype

Introduction

Linear fit method

Results with 1, 2, 3, and 5 GeV electrons

conclusions

Anne-Marie Magnan, IC London.

Introduction

- Complete test beam prototype : 30 layers, 1 cm^2 cells, 9 wafers per layer.
- Objective : determine position and angular resolution in test beam data, compared with the one obtained in MC simulation.
- Method : **linear fit ➔** take into account correlations between layers.
- For this study, only 1, 2, 3 and 5 GeV single electrons (DESY test beam).
- Own generation with Mokka05.05.

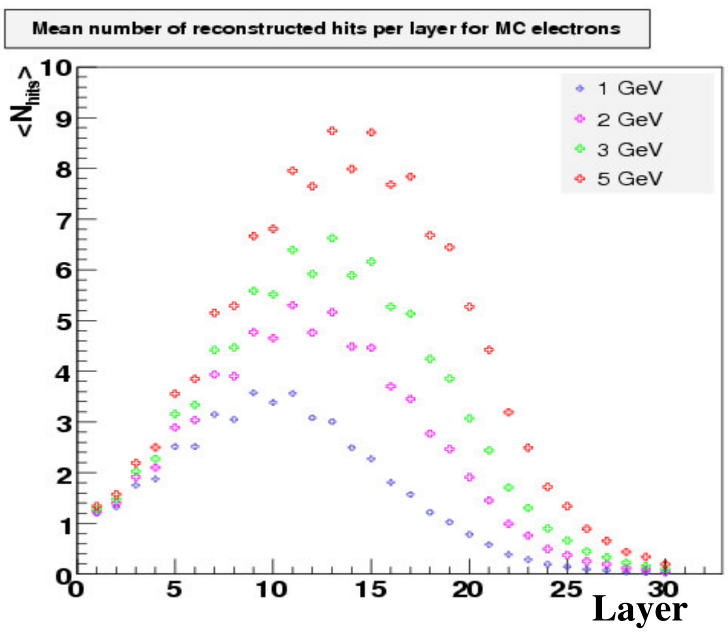
  Beam position and RMS : (0 ±10, 0 ± 10, -220 ± 0) (in mm).

  Current LCIO output does not allow to have the "truth" position in 1st ECAL layer after scattering in air/trackers materials.

Imperial College London

Linear fit method : definition of variables

- Definition of x and y position per layer :



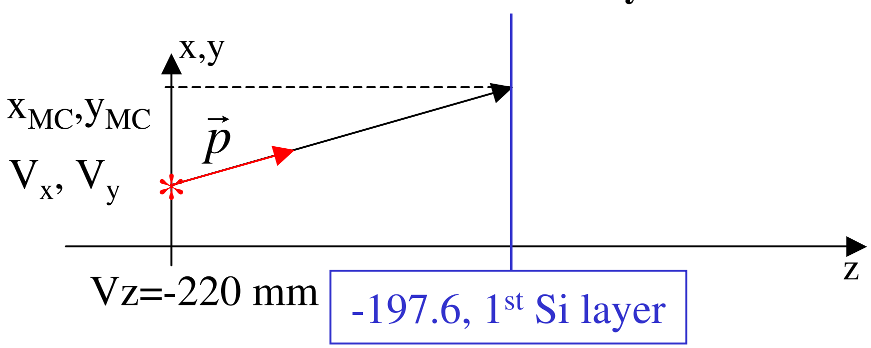
$$\bar{x} = \frac{\sum_i E_i x_i}{\sum_i E_i} \qquad \bar{y} = \frac{\sum_i E_i y_i}{\sum_i E_i}$$

i = hits in layer L.

<u>Variable of interest :</u>

reconstructed position compared to the expected one :

- $Dx = \bar{x} - x_{MC}$, $Dy = \bar{y} - y_{MC}$
- $x_{MC} = Vx + (z - Vz)\frac{\vec{p}_x}{\vec{p}_z}$, $y_{MC} = Vy + (z - Vz)\frac{\vec{p}_y}{\vec{p}_z}$



For this simulation : $\vec{p} = \vec{p}_z$

Redefine z = 0 ➔ 1st Si layer

Imperial College London

Linear fit method : definition of the χ^2

- Estimator of how accurate the prediction of the measurement is :
  - Without correlations between variables :

$$\chi^2 = \sum \frac{(x_{measured} - x_{theoretical})^2}{\sigma^2}$$

$$x = \begin{pmatrix} x \\ y \end{pmatrix}$$

  - With correlations between variables :

$$x_{theoretical} = p_{0x} + p_{1x} \times z$$

$$\chi^2 = \sum_{i,j} (x_{meas} - x_{th})_i W_{ij} (x_{meas} - x_{th})_j$$

i,j = 1,.....,30 for x
31,.....,60 for y

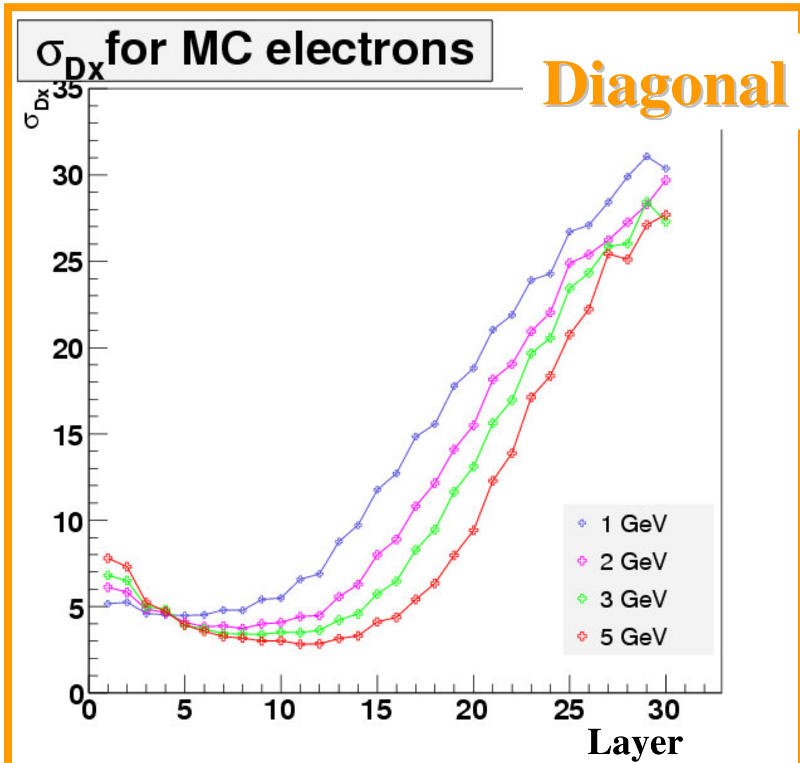
  - W_{ij} is the inverse of the error matrix E_{ij} :

$$E_{ij} = \mathrm{cov}(Dx_i, Dx_j) = \langle Dx_i Dx_j \rangle - \underbrace{\langle Dx_i \rangle \langle Dx_j \rangle}_{=0} = \langle Dx_i Dx_j \rangle$$

Imperial College London

Linear fit method : error matrix

o



Diagonal terms

x)

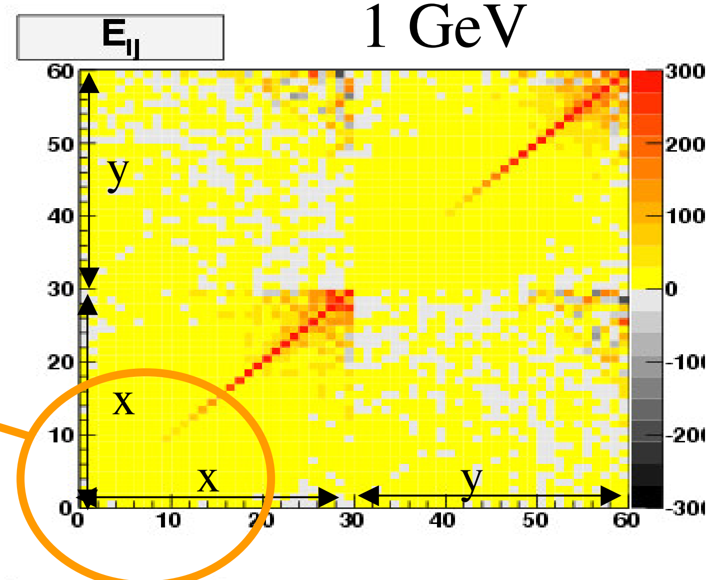
nt

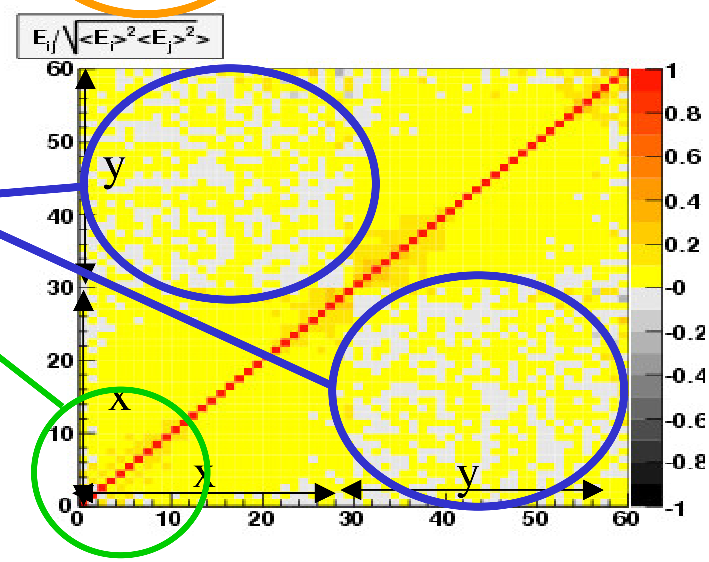
al values

between x and y

(= correlation matrix)

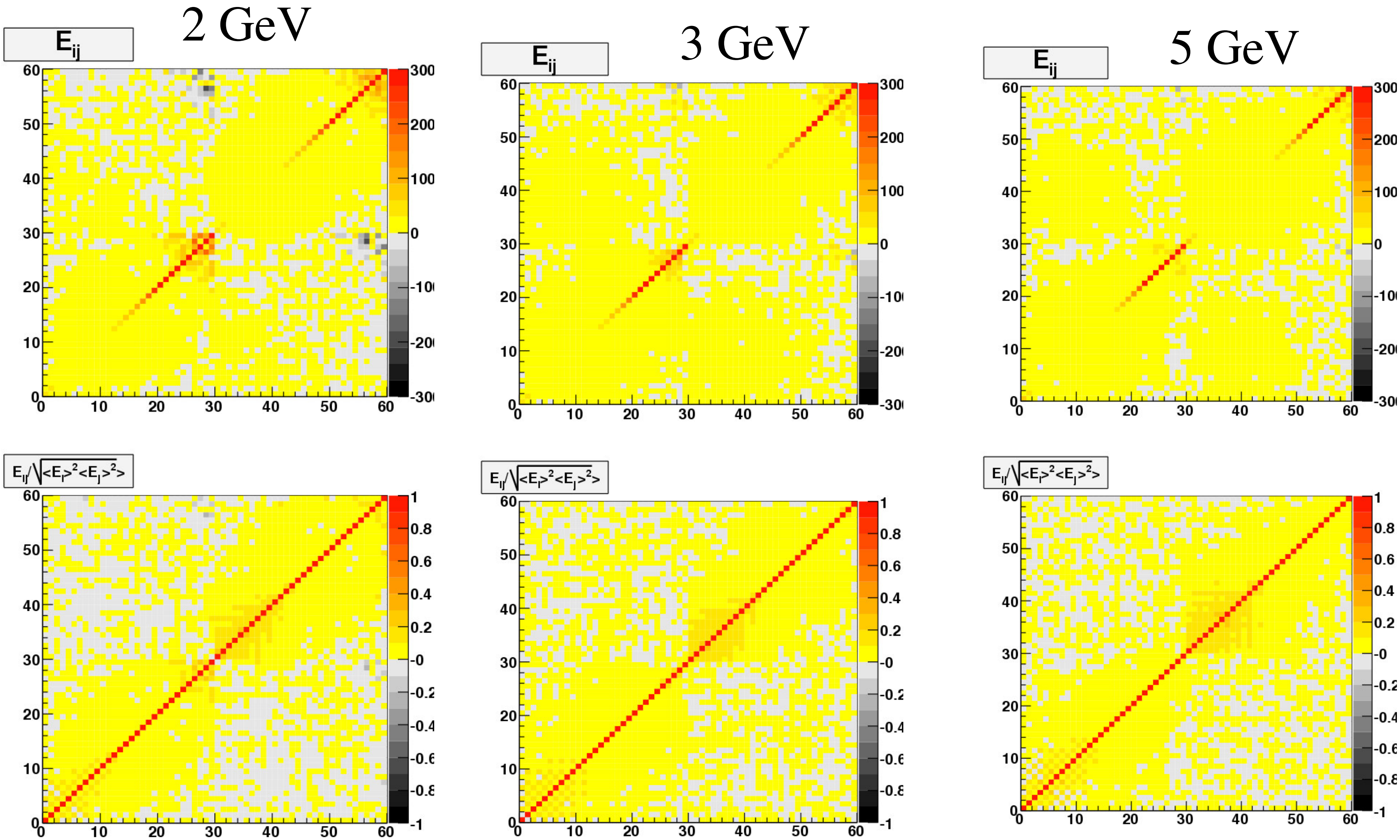
1 GeV





Funny pattern : due to non-overlapping layers in x ???

Error matrix for higher energies



Linear fit method : minimisation of the χ^2

- X and y are uncorrelated : we consider 2 (30,30) matrices

  ➔ 2 independent fits : one for x, the other for y.

  ➔ we can then look for the parameters (p_{0x},p_{1x}) of the linear fit which minimize the χ^2 :

$$\frac{\partial \chi^2}{\partial p_{1x}} = 0 \qquad \frac{\partial \chi^2}{\partial p_{0x}} = 0$$

x = x or y

- This gives the following equation :

$$\begin{pmatrix} W_{ij} & W_{ij} z_i \\ W_{ij} z_i & W_{ij} z_i z_j \end{pmatrix} \begin{pmatrix} p_{0x} \\ p_{1x} \end{pmatrix} = \begin{pmatrix} W_{ij} x_i \\ W_{ij} z_i x_j \end{pmatrix}$$

Reminder : $\chi^2 = \sum_{i,j} (x_{meas} - x_{th})_i W_{ij} (x_{meas} - x_{th})_j$

$$x_{theoretical} = p_{0x} + p_{1x} \times z$$

$$= \begin{pmatrix} \sigma_{p_{0x}}^2 & \rho \sigma_{p_{0x}} \sigma_{p_{1x}} \\ \rho \sigma_{p_{0x}} \sigma_{p_{1x}} & \sigma_{p_{1x}}^2 \end{pmatrix}$$

Imperial College
London

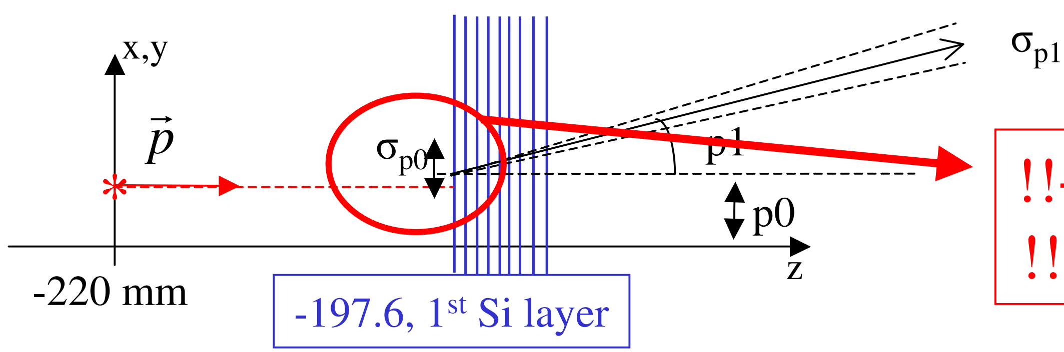
Linear fit method : expected resolution

Best case : if all layers

	σ_{p0x} (mm)	σ_{p0y} (mm)	σ_{p1x} (mrad)	σ_{p1y} (mrad)
1 GeV	[illegible]	[illegible]	[illegible]	[illegible]
2 GeV	[illegible]	[illegible]	[illegible]	[illegible]
3 GeV	[illegible]	[illegible]	[illegible]	[illegible]
5 GeV	2.2	2.5	35	37

Best case : if all layers

- Angular resolution decrease when E increase : more layers on the back ➔ better
  [illegible] e as [illegible] hits ➔ better estimate of the average position
- Position resolution is higher in y , why ???????



!!+ tracker resolution!!
!! in data !!

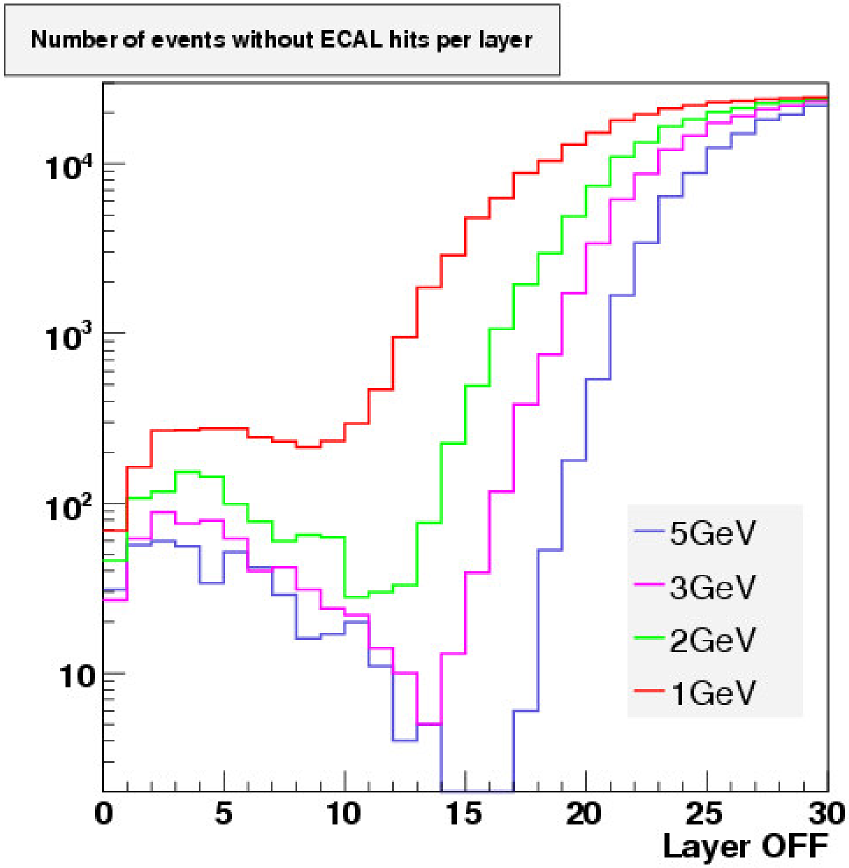
Imperial College London

Position and angular resolution obtained on an event by event basis

Equation to solve :

$$\begin{pmatrix} W_{ij} & W_{ij}z_i \\ W_{ij}z_i & W_{ij}z_iz_j \end{pmatrix}\begin{pmatrix} p_{0x} \\ p_{1x} \end{pmatrix} = \begin{pmatrix} W_{ij}\bar{x}_i \\ W_{ij}z_i\bar{x}_j \end{pmatrix}$$

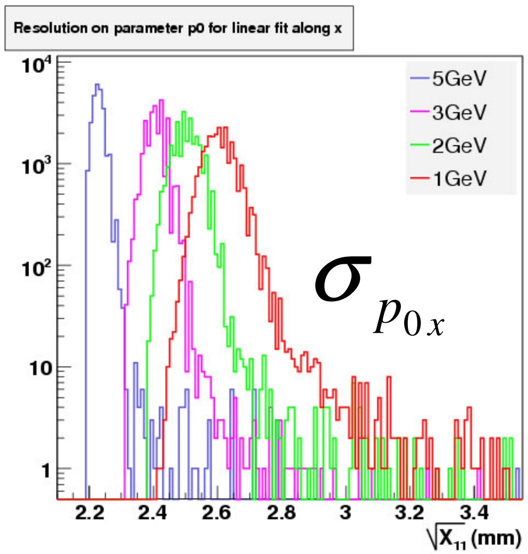
To solve this, need to take into account only layers i and j with hits ➔ remove layers with no hit from error matrix, then invert to have W matrix.

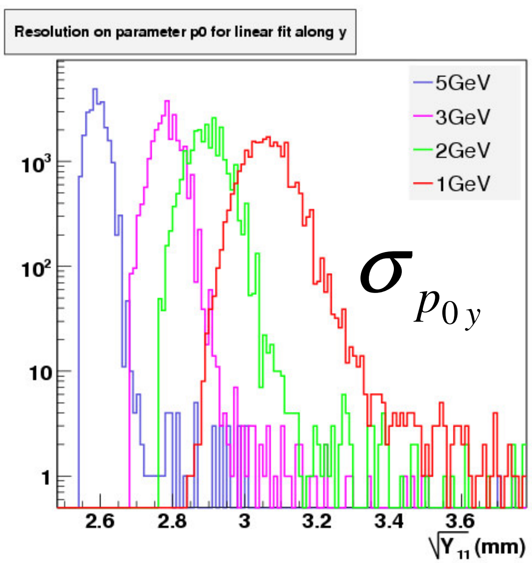


- ➔ Therefore have to **solve it event by event**.

Imperial College
London

Results event by event
for parameter resolution matrices

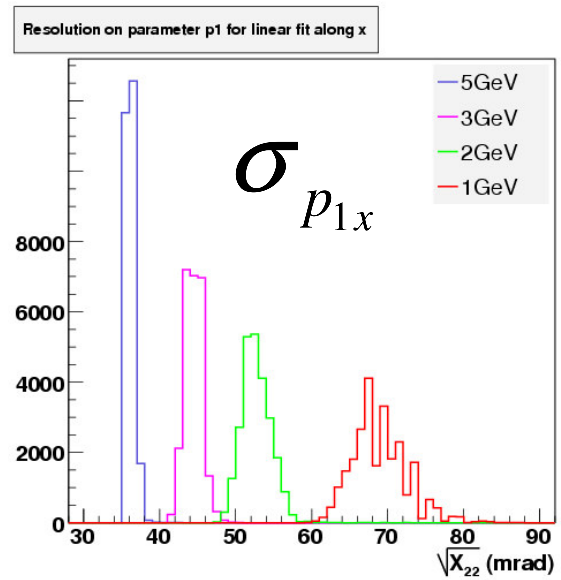


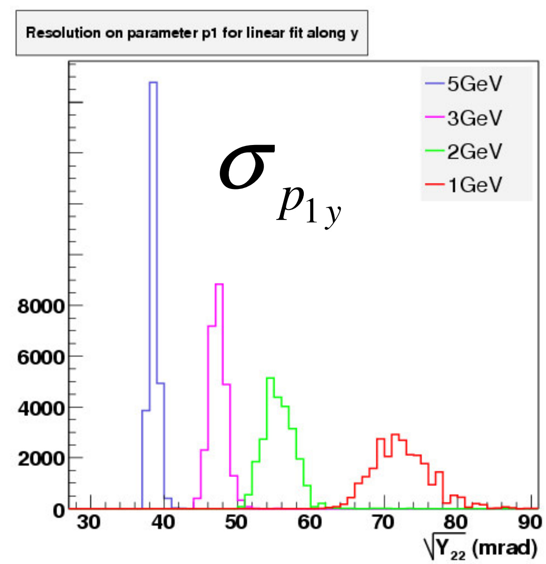


$$\begin{pmatrix} W_{ij} & W_{ij}z_i \\ W_{ij}z_i & W_{ij}z_iz_j \end{pmatrix}^{-1} \begin{pmatrix} p_{0x} \\ p_{1x} \end{pmatrix} = \begin{pmatrix} W_{ij}\bar{x}_i \\ W_{ij}z_i\bar{x}_j \end{pmatrix}$$

=

$$\begin{pmatrix} \sigma_{p_{0x}}^{\ 2} & \rho\sigma_{p_{0x}}\sigma_{p_{1x}} \\ \rho\sigma_{p_{0x}}\sigma_{p_{1x}} & \sigma_{p_{1x}}^{\ 2} \end{pmatrix}$$



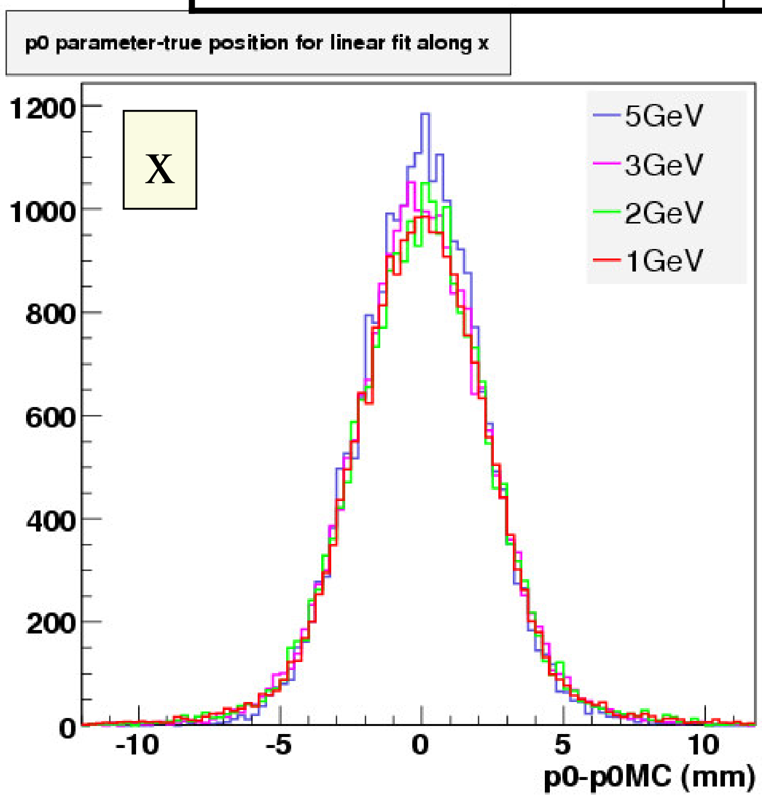


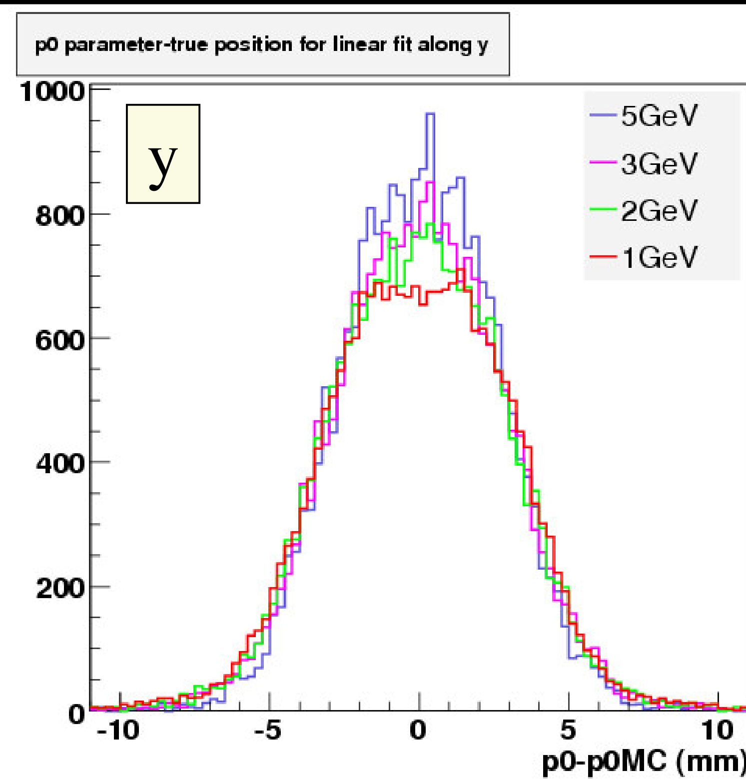
Imperial College London

Result event by event

for $(p_0 - p_{0MC})_{x,y}$

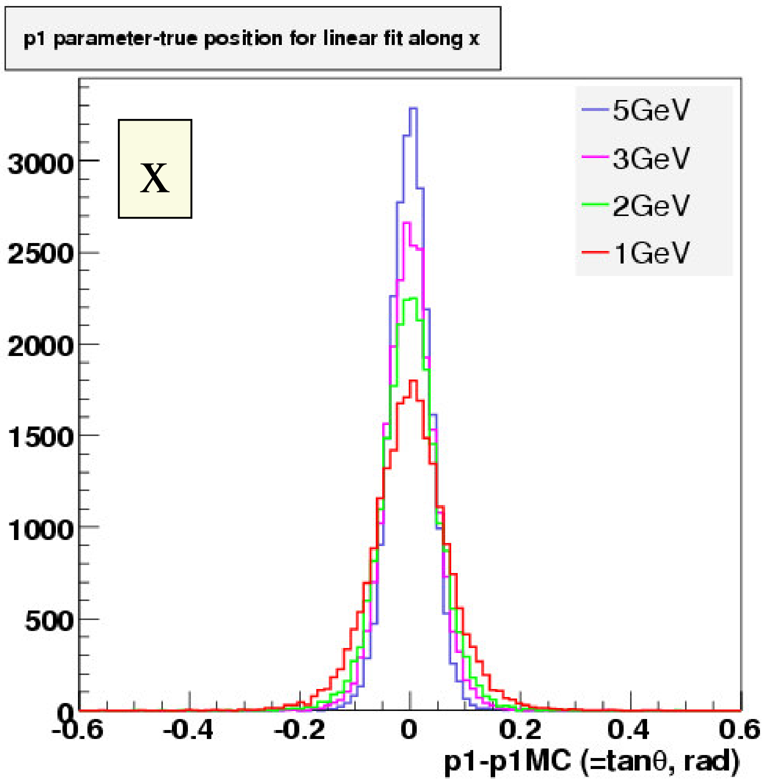
Energy	σ_{p0x} (mm)	if all layers	σ_{p0y} (mm)	if all layers
1 GeV	2.6	2.4	3.1	2.8
2 GeV	2.5	2.4	2.9	2.8
3 GeV	2.4	2.3	2.8	2.7
5 GeV	2.2	2.2	2.6	2.5

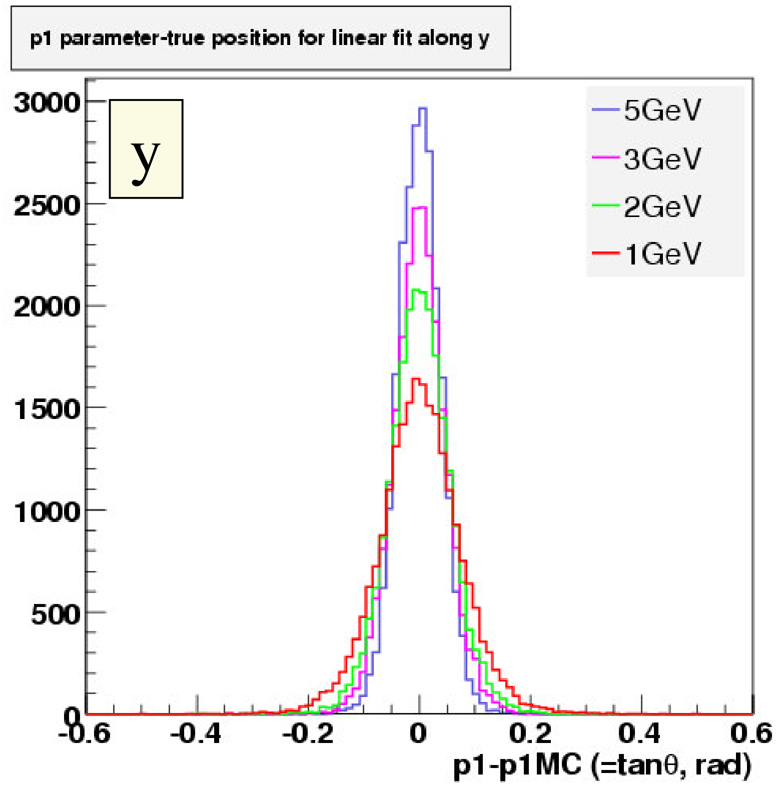




Result event by event for $(p_1 - p_{1MC})_{x,y}$

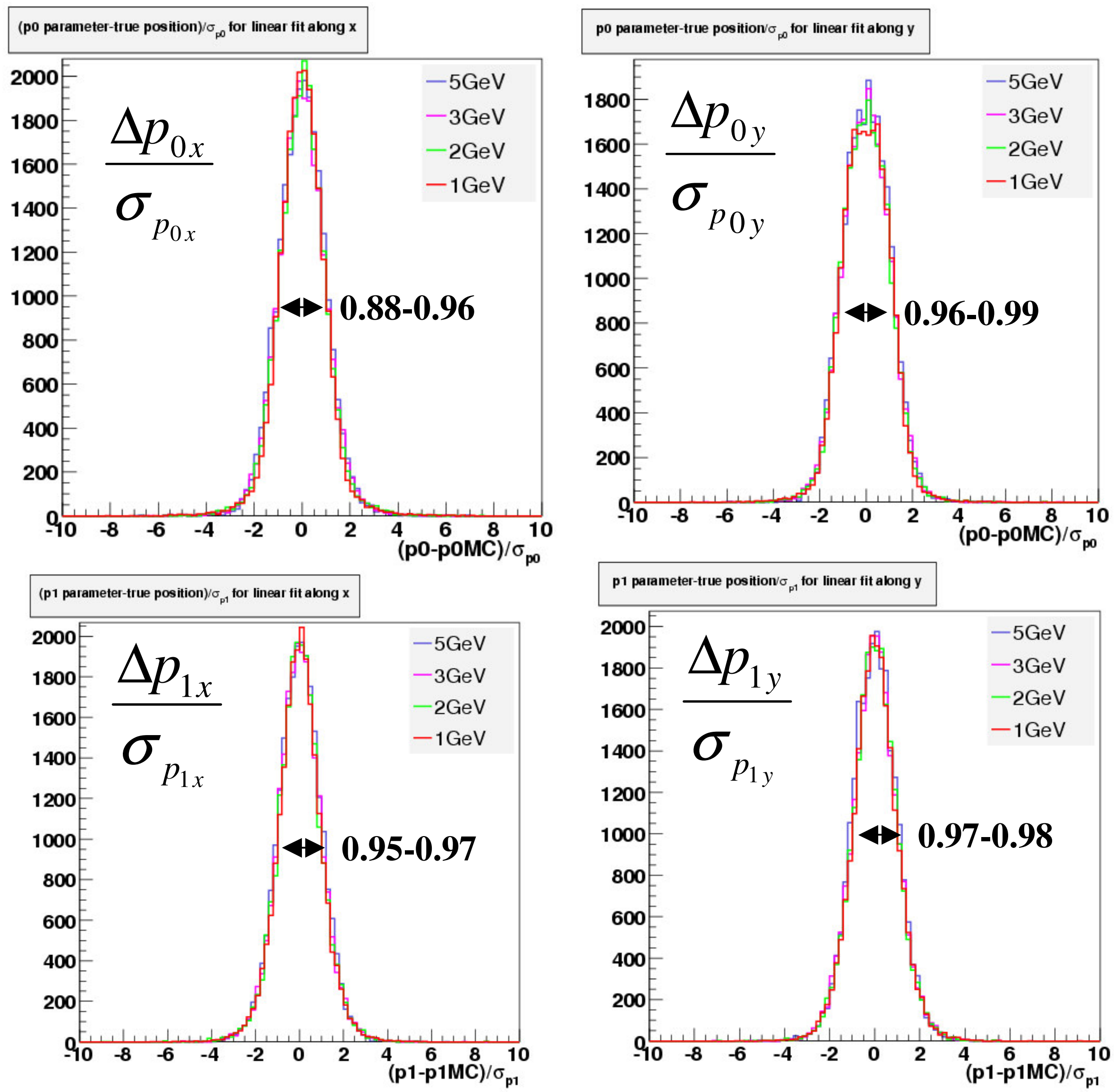
Energy	σ_{p1x} (mrad)	if all layers	σ_{p1y} (mrad)	if all layers
1 GeV	71	58	74	60
2 GeV	54	48	56	50
3 GeV	45	41	48	44
5 GeV	36	35	39	37





Consistency checks

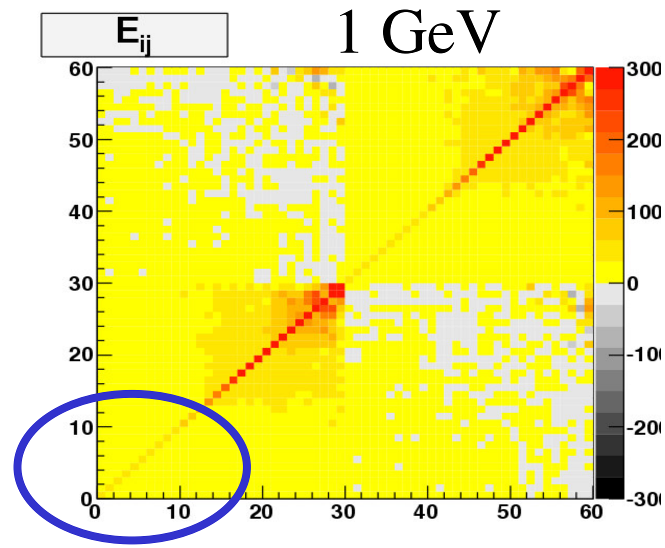
- Pull of the distributions for Δp_0 ($=p_0 - p_{0MC}$) and Δp_1

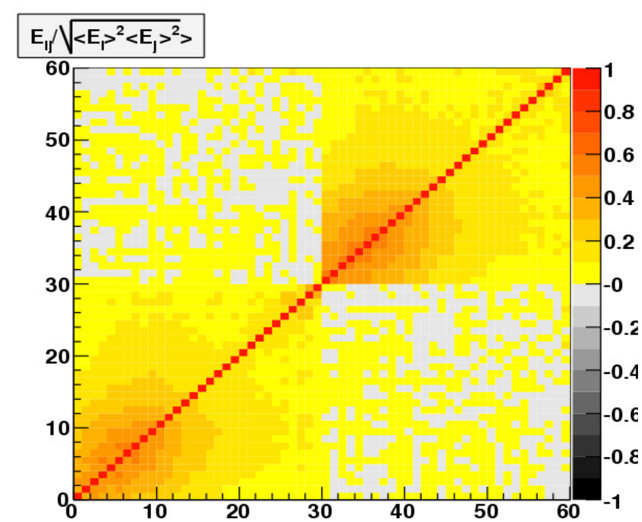


Imperial College London

With material in front of ECAL

- Beam position : (4,7,10000) mm
- Expected effect of air scattering in 10 m ➔ ~ 13 mm spread.
- Observed <x> : ~ 16 mm spread.
- Expected resolution :
  - The “true” position is now given by hits in last DC layer
  - σ_{p0x} = 5.2 mm, σ_{p0y} = 5.3 mm
  - σ_{p1x} = 70 mrad, σ_{p1y} = 69 mrad
  - ➔ still correlations. Need to have the “true” position of MC particle at front ECAL face.





Imperial College London

Hakan + AM

Future work

- Study with missing layers : better to have front, middle, back ? First layers needed for position resolution, and last ones for angular resolution… but depend on energy.

- Redo everything with material in front, and truth entry point.

- Study of reconstructed tracking resolution to separate the 2 sources and allow to compare with data.

- Redo everything when realistic digitisation is available.

Imperial College
London

Thank you for your attention