



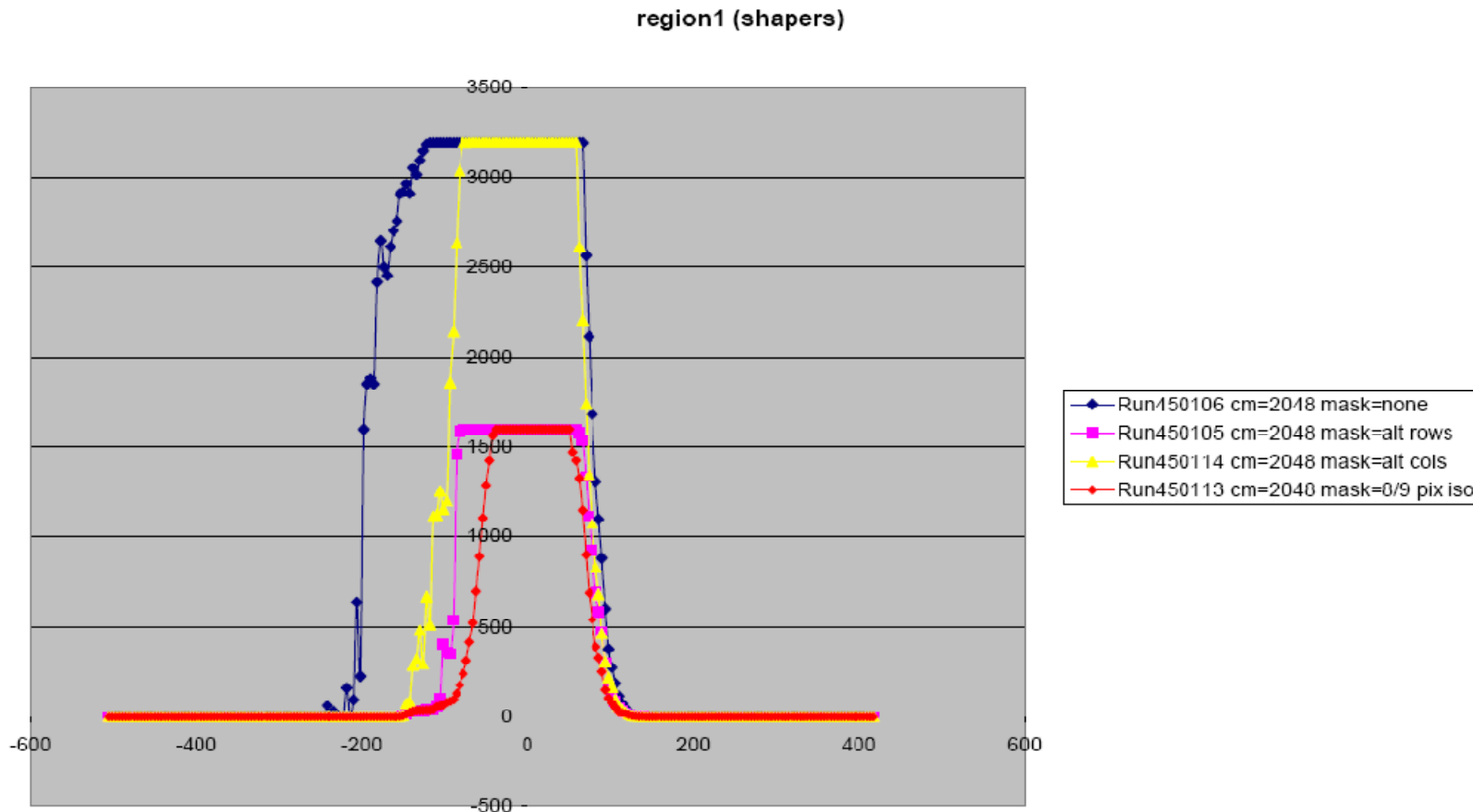
Science & Technology
Facilities Council

Using Hit Frequency curves

Calice meeting
R. Turchetta

29 February 2008

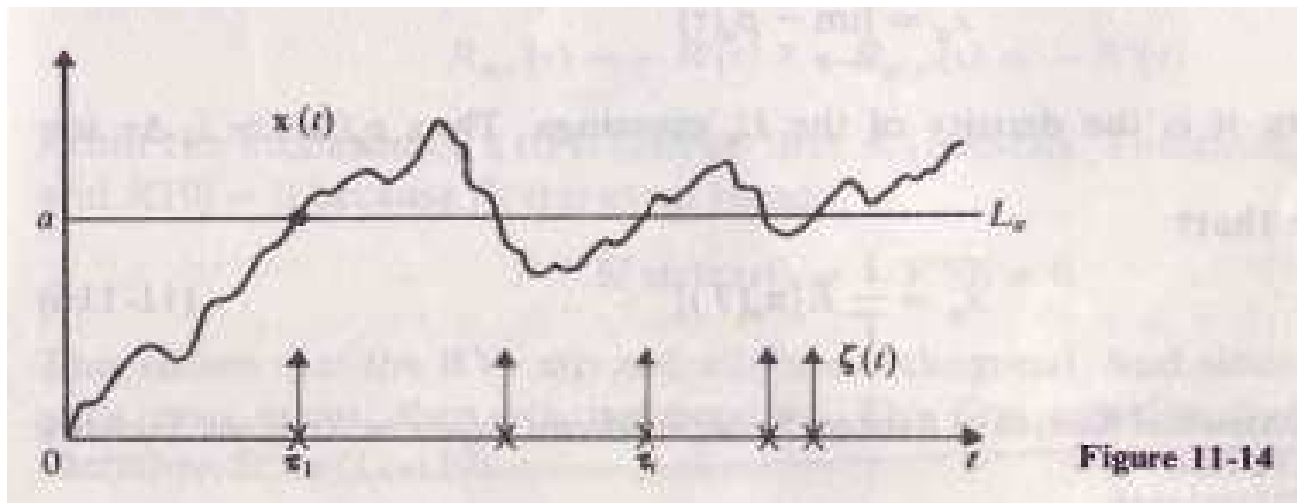
Can we use these curves to extract quantitative information about noise, gain, ...



Level-crossing problem.

Random variable $x(t)$, crossing a level set at a .

How often does a crossing occur?



See: A. Papoulis, *Probability, Random Variables and Stochastic Processes*, section 11.4



General result

If $\mathbf{x}(t)$ is differentiable, the level-crossing density λ_a (i.e. the average number of crossings in unit time) is

$$\lambda_a = f_{\mathbf{x}}(a) \cdot E\left\{|\mathbf{x}'(t)| \mid \mathbf{x}(t) = a\right\}$$

Where $f_{\mathbf{x}}(a)$ denotes the probability density for \mathbf{x} and $E\{\dots\}$ indicates expectation values.



Gaussian distribution

For a normal process with zero mean (if this is not true, just apply a shift), the theory yields

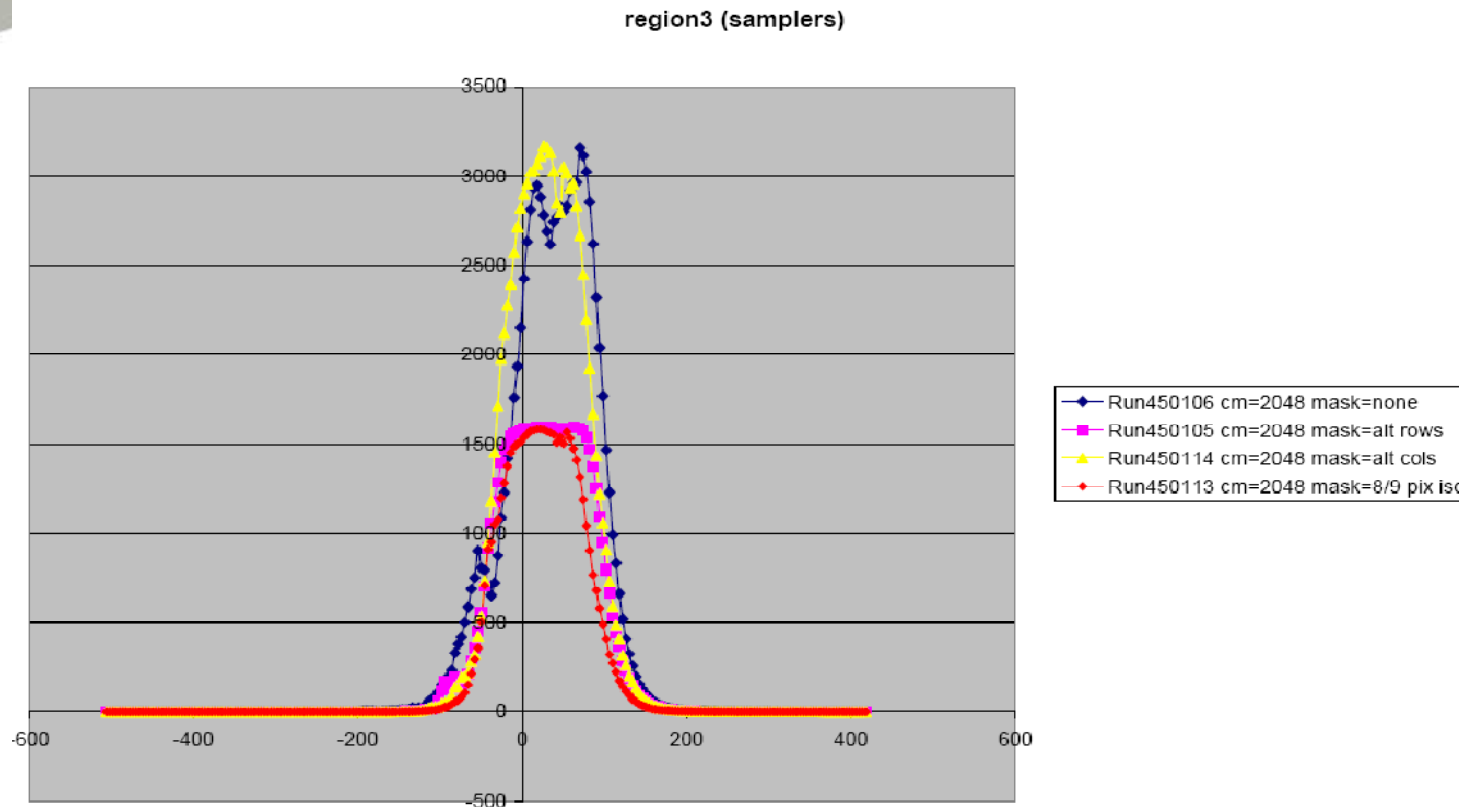
$$\lambda_a = \frac{1}{\pi} \sqrt{\frac{-R''(0)}{R(0)}} \exp\left[-\frac{a^2}{2R(0)}\right]$$

where $R(\tau)$ is the autocorrelation function for $\mathbf{x}(t)$.

$R(0) = E\{\mathbf{x}(t)\mathbf{x}(t)\} = \text{variance of the process}$



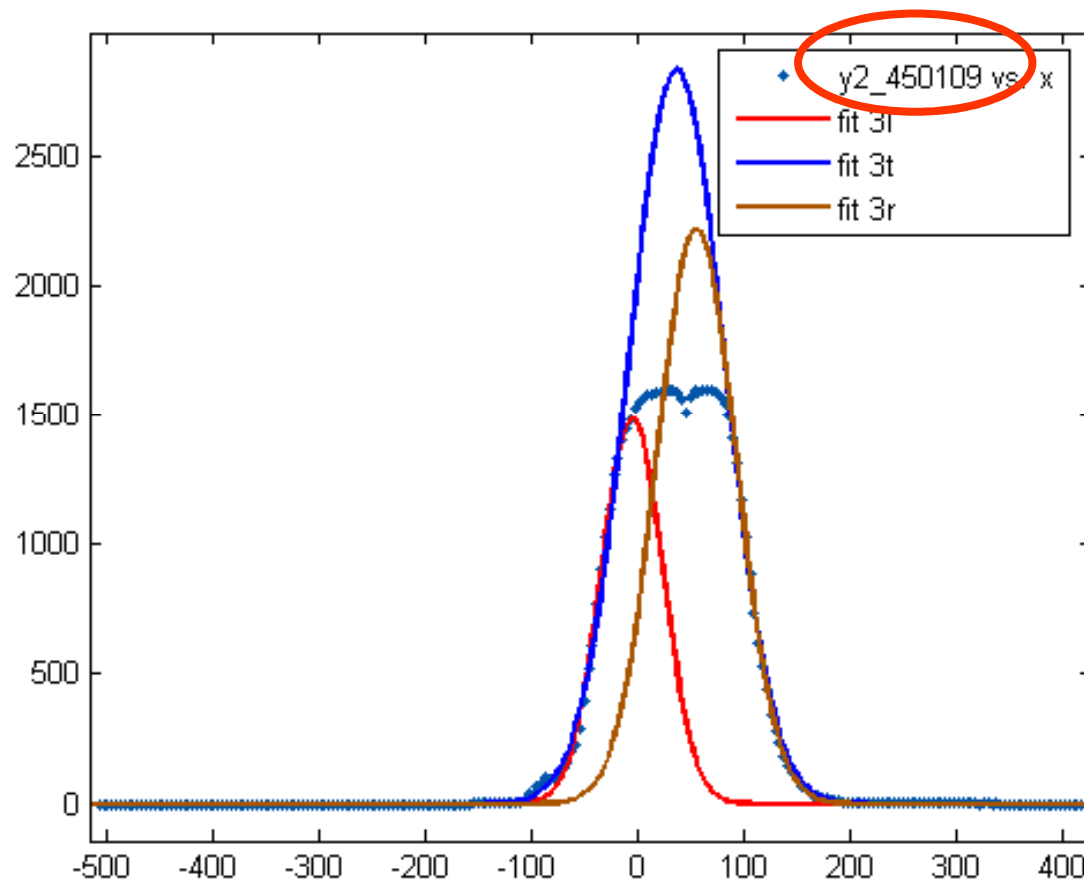
Example



It looks nice and Gaussian, but ...
close to zero, memory overflow plus bandwidth
limitations



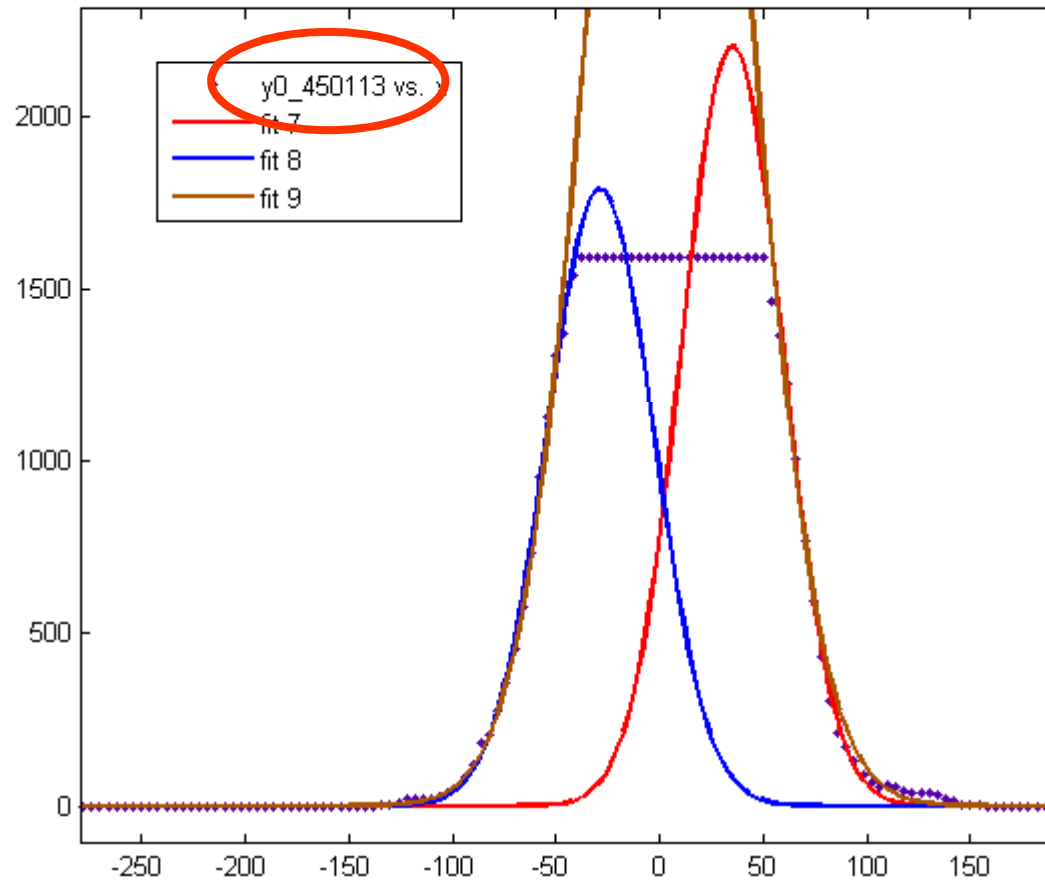
Example. 1



Fit on one or the other side or global.



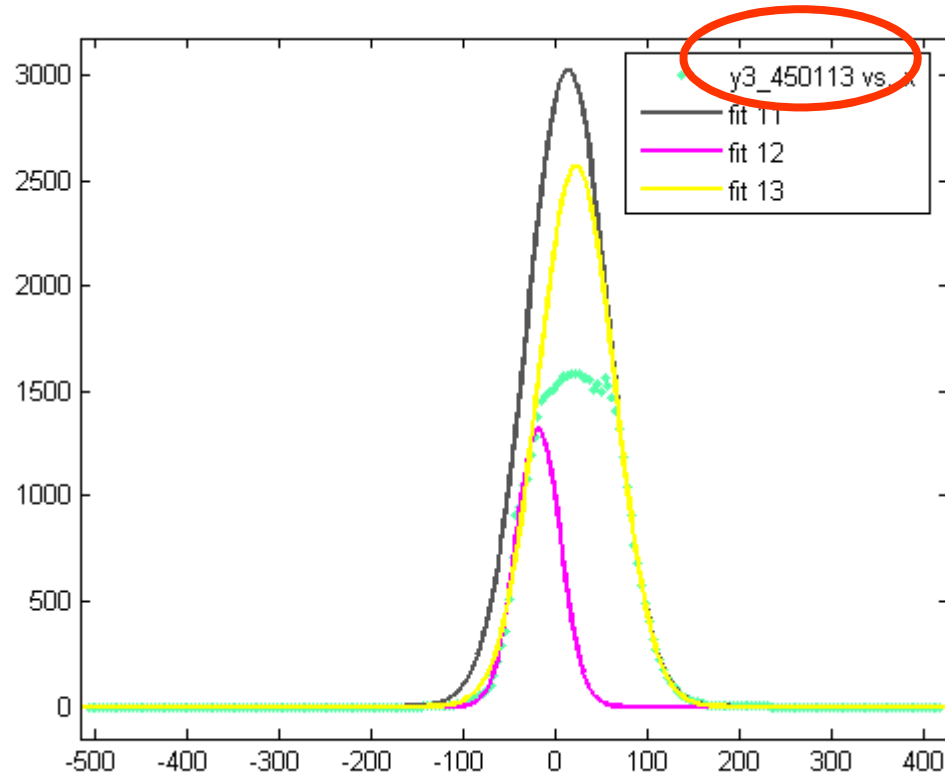
Example. 2



Fit on one or the other side or global.



Example. 3



Fit on one or the other side or global.