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On the non-vanishing of the Collins mechanism for single spin asymmetries

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Abstract

The Collins mechanism provides a non-perturbative explanation for the large single-spin asymmetries found in hard semi-inclusive reactions involving a transversely polarized nucleon. However, there are seemingly convincing reasons to suspect that the mechanism vanishes, and indeed it does vanish in the naive parton model where a quark is regarded as an essentially "free" particle. We give an intuitive analysis which highlights the difference between the naive picture and the realistic one and shows how the Collins mechanism arises when the quark is described as an off-shell particle by a field in interaction.

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I. INTRODUCTION

The Collins mechanism [1] provides a possible explanation of the very large single-spin asymmetries – sometimes as big as 40% – found in semi-inclusive reactions like $p^{\uparrow}p \to \pi X$ or $ep^{\uparrow} \to e\pi X$, where $p^{\uparrow}$ is a transversely polarized proton. The existence of these asymmetries has been known for decades and one of the great puzzles has been the fact that it is quite impossible to produce such large asymmetries via perturbative QCD mechanisms. The Collins mechanism is a "soft" non-perturbative effect describing the fragmentation of a transversely polarized quark into e.g. $\pi + X$. It makes the angular distribution of the pion with respect to the direction of motion of the quark depend upon the direction of the quark's transverse polarization. But this feature is itself puzzling, since it is well known [2, 3] that in the decay of a transversely polarized particle or resonance $R^{\uparrow} \to \pi + X$ the angular distribution $W(\theta, \phi)$ of the pion cannot depend on the direction of the transverse polarization of R , in any parity conserving theory. Indeed, as we shall see explicitly, the Collins mechanism vanishes in the naive parton model where the quark is visualized as a "free" particle. The Collins effect thus depends crucially on treating the quark as off-shell and described by an interacting field.

It was at one time thought that the time-reversal invariance of QCD could be utilized to prove the vanishing of the Collins mechanism, but Collins [1] pointed out a flaw in the argument based upon a failure to distinguish between so-called *in* and *out* states in field theory. Although this mathematical argument shows that the Collins mechanism need not vanish, it does not provide an intuitive picture of what is happening.

In this paper we give an intuitive analysis of the Collins mechanism based upon rotational invariance, which makes clear the difference between particle or resonance decay, which *cannot* have a Collins effect, and quark fragmentation, which can.

In Section 2 we revisit the standard QCD picture for hard semi-inclusive reactions, stressing the difference between treating the active quark as "free" and as an off-shell interacting particle. Section 3 derives the angular distribution, contrasting the cases of "free" and interacting quarks. The results are discussed in Section 4. Some technical details are relegated to an Appendix.

II. QCD ANALYSIS OF $\mathbf{e\,p^{\uparrow} \rightarrow e\,\pi\,X}$

For concreteness we shall present our analysis in the context of semi-inclusive deep inelastic lepton-proton scattering (see Fig. 1) in which an unpolarized lepton interacts with a transversely polarized proton and the final state pion is detected at angles (θ, ϕ) in the"γ" p CM frame, with the proton moving along the OZ axis.

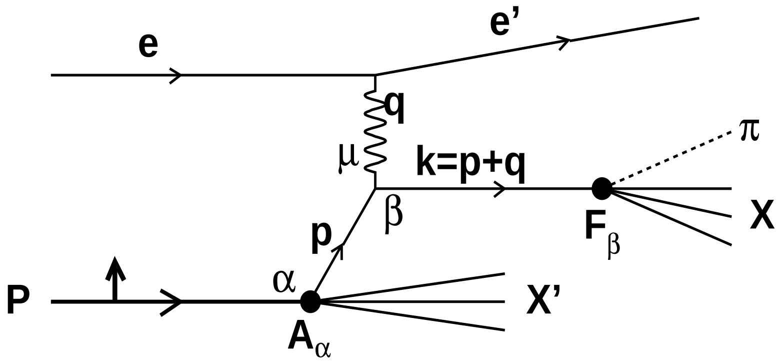

FIG. 1: Feynman diagram for semi-inclusive scattering

Regarding Fig. 1 as a Feynman diagram, the amplitude for the hadronic part of the process is given by

$$H^{\mu} = F_{\beta}(\gamma^{\mu})_{\beta\alpha}A_{\alpha} \tag{1}$$

where A_{α} is the amplitude for the proton, momentum P, to emit a quark with four-momentum p and Dirac index α, and F_{β} is the amplitude for a quark of four-momentum k and Dirac index β to fragment into a pion and some other particle X. As is usual in the parton model, both A_{α} and F_{β} are defined to include their adjacent quark propagators. In QCD these amplitudes are given by:

$$A_{\alpha} = \int d^4x e^{ip.x} < X' \mid \Psi_{\alpha}(x) \mid P > \tag{2}$$

$$F_{\beta} = \int d^4z e^{-ik.z} < \pi, X \mid \bar{\Psi}_{\beta}(z) \mid 0 > \tag{3}$$

where k_{μ} is a time-like four-vector, $k^0 > 0$, and the hadronic tensor $W^{\mu\nu}$ is then

$$W^{\mu\nu} = \sum_{X,X'} H^{\mu} H^{\nu *} \tag{4}$$

Note that the πX final state is, strictly speaking, an *out* state, but this is irrelevant in our analysis.

In these and all further expressions we are only interested in the general structure of the amplitudes, so we will ignore numerical and other inessential factors Also, since we deal with unpolarized leptons, we are only concerned with the part of $W^{\mu\nu}$ symmetric under $\mu \leftrightarrow \nu$.

Substituting Eq. (1) into Eq. (4) we obtain

$$W^{\mu\nu} = \rho^{\mu\nu}_{\alpha\beta} \sum_{X} F_{\alpha} \bar{F}_{\beta} \tag{5}$$

where, as usual,

$$\bar{F}_{\beta} = F^{*}_{\tau} \gamma^{0}_{\tau\beta} \tag{6}$$

and

$$\rho^{\mu\nu}_{\alpha\beta} = (\gamma^{\mu} \Phi \gamma^{\nu})_{\alpha\beta} \tag{7}$$

with Φ the usual quark-quark correlation function

$$\Phi_{\alpha\beta} = \int d^4x e^{-ip.x} < P \mid \bar{\Psi}_{\beta}(x) \Psi_{\alpha}(0) \mid P > \tag{8}$$

Since the labels μ, ν play no role in the following, aside from the symmetry under $\mu \leftrightarrow \nu$, we shall suppress them in the following, simply writing the LHS of Eq. (7) as $\rho_{\alpha\beta}$.

Consider now the structure of the $q \to \pi X$ fragmentation amplitude

$$\begin{aligned} F_{\beta} &\equiv F_{\beta}(p_{\pi}; X, \lambda_X; k) \\ &= \int d^4z e^{-ik.z} < p_{\pi}; X, \lambda_X \mid \bar{\Psi}_{\beta}(z) \mid 0 > \end{aligned} \tag{9}$$

where λ_X is the helicity of X.

If $\bar{\Psi}(z)$ were a free field $\bar{\Psi}^0(z)$ we could make the usual Fourier expansion into creation and annihilation operators

$$
\begin{aligned}
\bar{\Psi}^0_\beta(z) \;&\sim\; \sum_\lambda \int \frac{d^3p}{E_p}\{\bar{u}_\beta(\boldsymbol{p},\lambda)a_0^\dagger(\boldsymbol{p},\lambda)e^{ip.z} \\
&+\; \bar{v}_\beta(\boldsymbol{p},\lambda)b_0(\boldsymbol{p},\lambda)e^{-ip.z}\}
\end{aligned} \tag{10}
$$

where $E_p = \sqrt{\boldsymbol{p}^2 + m^2}$, and the subscript 0 on the operators signifies free field operators. When $\bar{\Psi}(z)$ is a field in interaction we can still make a 3-dimensional Fourier expansion at some instant of time, say $t = 0$. Then one finds [4], at arbitrary times t,

$$
\begin{aligned}
\bar{\Psi}_\beta(z) \;&\sim\; \sum_\lambda \int \frac{d^3p}{E_p}\{\bar{u}_\beta(\boldsymbol{p},\lambda)a^\dagger(\boldsymbol{p},\lambda;t)e^{-i\boldsymbol{p}.\boldsymbol{z}} \\
&+\; \bar{v}_\beta(\boldsymbol{p},\lambda)b(\boldsymbol{p},\lambda;t)e^{i\boldsymbol{p}.\boldsymbol{z}}\}
\end{aligned} \tag{11}
$$

where now the time dependence is non-trivial, being controlled by the Hamiltonian H:

$$
\begin{aligned}
a^\dagger(\boldsymbol{p},\lambda;t) &= e^{iHt}a^\dagger(\boldsymbol{p},\lambda)e^{-iHt} \\
b(\boldsymbol{p},\lambda;t) &= e^{iHt}b(\boldsymbol{p},\lambda)e^{-iHt}
\end{aligned} \tag{12}
$$

This is to be contrasted with the free field case where

$$
\begin{aligned}
a_0^\dagger(\boldsymbol{p},\lambda;t) &= e^{iE_pt}a_0^\dagger(\boldsymbol{p},\lambda) \\
b_0(\boldsymbol{p},\lambda;t) &= e^{-iE_pt}b_0(\boldsymbol{p},\lambda)
\end{aligned} \tag{13}
$$

Of course the interacting $a^\dagger(\boldsymbol{p},\lambda;t)$ and $b(\boldsymbol{p},\lambda;t)$ are not simply one-particle creation and annihilation operators. However, the spinors occurring in Eqs. (10) and (11) are the same as in the free case i.e. are mass m spinors.

Bearing in mind Eq. (11) we have for the field operator appearing in Eqs. (3) and (9)

$$
\begin{aligned}
\bar{\Psi}_\beta(k) \equiv \int d^4z e^{-ik.z}\bar{\Psi}_\beta(z) \;&=\; \frac{1}{E_k}\sum_\lambda\{\bar{u}_\beta(\boldsymbol{k},\lambda)A^\dagger(k,\lambda) \\
&+\; \bar{v}_\beta(-\boldsymbol{k},\lambda)B(k,\lambda)\}
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
A^\dagger(k,\lambda) &\equiv \int dte^{-ik_0t}a^\dagger(\boldsymbol{k},\lambda;t) \\
B(k,\lambda) &\equiv \int dte^{-ik_0t}b(-\boldsymbol{k},\lambda;t)
\end{aligned}
\tag{15}
$$

As stressed earlier, these are not single particle creation and annihilation operators. However, acting on the vacuum state $\mid 0 >$ they produce states which, as is shown in the Appendix, are eigenstates of momentum and energy with definite helicity and definite parity. Thus we shall define

$$
\mid k^\mu,\lambda)_+ \equiv A^\dagger(k,\lambda) \mid 0 > \tag{16}
$$

and

$$
\mid k^\mu,\lambda)_- \equiv B(k,\lambda) \mid 0 > \tag{17}
$$

and the key point will be the fact that the states in Eqs. (16) and (17) have *opposite* parity, as indicated by the labels +/-. We use the notation $\mid\quad)$ to emphasize the fact that these states are not necessarily one particle states. Hence we may write Eq. (9) in the form

$$
\begin{aligned}
F_\beta(p_\pi;X,\lambda_X;k) = \frac{1}{E_k}\sum_\lambda \{ &\bar{u}_\beta(\boldsymbol{k},\lambda) < p_\pi;X,\lambda_X \mid k^\mu,\lambda)_+ \\
&\bar{v}_\beta(-\boldsymbol{k},\lambda) < p_\pi;X,\lambda_X \mid k^\mu,\lambda)_- \}
\end{aligned}
\tag{18}
$$

This is to be contrasted with the expression one obtains for the decay of a *particle* of momentum $\boldsymbol{k}$ (or for the case of a free quark field) namely

$$
F^0_\beta(p_\pi;X,\lambda_X;k) = \frac{1}{E_k}\sum_\lambda \bar{u}_\beta(\boldsymbol{k},\lambda) < p_\pi;X,\lambda_X \mid k,\lambda > \tag{19}
$$

The analogue of the second term on the RHS of Eq.(18) is here absent, because, via Eq. (10), it involves

$$
\int dte^{-ik_0t}e^{-iE_pt} \propto \delta(k_0+E_p) = 0 \tag{20}
$$

since $k_0 > 0$.

III. THE HADRON ANGULAR DISTRIBUTION

It will suffice to consider the decay

$$\text{“}q\text{”} \longrightarrow \pi + X$$

in the CM system of the π and X i.e. where $\boldsymbol{p}_\pi + \boldsymbol{p}_X = 0$, since using the properties of the states $\mid k, \lambda)_{+/-}$ derived in the Appendix one can then derive the angular distribution in any other frame.

Let us look first at the decay of a spin1/2 particle (equivalently, the case of a “free” quark) where Eq. (19) holds. From Eq. (5) and (19)

$$W^{\mu\nu} \propto \rho^{uu}_{\lambda'\lambda} \sum_X < p_\pi; X, \lambda_X \mid k, \lambda >< p_\pi; X, \lambda_X \mid k, \lambda' >^* \tag{21}$$

where

$$\rho^{uu}_{\lambda'\lambda} = \bar{u}_\alpha(k, \lambda')\rho_{\alpha\beta}u_\beta(k, \lambda) \tag{22}$$

Rotational and parity invariance allow us to write, for the matrix element in Eq. (21), in the frame where $\boldsymbol{p}_\pi = -\boldsymbol{p}_X$

$$< p_\pi; X, \lambda_X \mid k, \lambda >= M(\lambda_X)e^{i\lambda\phi}d^{1/2}_{\lambda\lambda_X}(\theta) \tag{23}$$

where (θ, ϕ) are the polar angles of $\boldsymbol{p}_\pi$ and the reduced matrix element $M(\lambda_X)$ is independent of λ [3], and

$$M(-\lambda_X) = \eta M(\lambda_X) \qquad (\eta = \pm 1) \tag{24}$$

Then Eq. (21) becomes

$$W^{\mu\nu} \propto \rho^{uu}_{\lambda'\lambda}e^{i\phi(\lambda'-\lambda)} \sum_{\lambda_X} \mid M(\lambda_X) \mid^2 d^{1/2}_{\lambda'\lambda_X}(\theta)d^{1/2}_{\lambda\lambda_X}(\theta) \tag{25}$$

For a spin1/2 particle moving along OZ the dependence on the transverse polarization is contained in the off diagonal helicity matrix elements

$$\rho^{uu}_{+-} = \rho^{uu^*}_{-+} = \frac{1}{2}(P_x - iP_y) \tag{26}$$

The contribution from ρ^{uu}_{+-} to Eq. (25) is

$$\rho^{uu}_{+-}e^{i\phi}\{\mid M(1/2)\mid^2 d^{1/2}_{1/2,1/2}(\theta)d^{1/2}_{-1/2,1/2}(\theta) + \mid M(-1/2)\mid^2 d^{1/2}_{1/2,-1/2}(\theta)d^{1/2}_{-1/2,-1/2}(\theta)\} \tag{27}$$

$$= \rho^{uu}_{+-}e^{i\phi}\mid M(1/2)\mid^2 \{d^{1/2}_{1/2,1/2}(\theta)d^{1/2}_{-1/2,1/2}(\theta) + d^{1/2}_{-1/2,-1/2}(\theta)d^{1/2}_{1/2,-1/2}(\theta)\} \tag{28}$$

where we have used Eq. (24).

Now

$$d^J_{-1/2,-1/2}(\theta) = d^J_{1/2,1/2}(\theta), \qquad d^J_{1/2,-1/2}(\theta) = -d^J_{-1/2.1/2}(\theta) \tag{29}$$

so that the expression in parenthesis in Eq. (28) vanishes. Similarly the term arising from ρ^{uu}_{-+} vanishes.

Hence, in the decay of a particle or a "free" quark "q" $\to \pi + X$, the angular distribution of the pion is independent of the transverse polarization of the decaying particle, i.e. the Collins mechanism vanishes.

The crucial point leading to the above conclusion was our ability to factor out the reduced matrix element terms $\mid M(1/2)\mid^2 = \mid M(-1/2)\mid^2$ in going from Eq. (27) to Eq. (28). It is this step that fails in the case of an interacting quark field. For the the analogoue of Eq. (21) is, on putting Eq. (18) into Eq. (5),

$$W^{\mu\nu} \propto \rho^{uu}_{\lambda'\lambda}\sum_{\lambda_X}\mathcal{M}_+\mathcal{M}'^*_+ + \rho^{vv}_{\lambda'\lambda}\sum_{\lambda_X}\mathcal{M}_-\mathcal{M}'^*_- + \rho^{uv}_{\lambda'\lambda}\sum_{\lambda_X}\mathcal{M}_-\mathcal{M}'^*_+ + \rho^{vu}_{\lambda'\lambda}\mathcal{M}_+\mathcal{M}'^*_- \tag{30}$$

where, for brevity we have used

$$\begin{aligned}\mathcal{M}_{+/-} &\equiv < p_\pi; X, \lambda_X \mid k, \lambda)_{+/-} \\ \mathcal{M}'_{+/-} &\equiv < p_\pi; X, \lambda_X \mid k, \lambda')_{+/-}\end{aligned} \tag{31}$$

and where, analogously to Eq. (22),

$$\begin{aligned}\rho^{vv}_{\lambda'\lambda} &= \bar{v}_\alpha(k,\lambda')\rho_{\alpha\beta}v_\beta(k,\lambda) \\ \rho^{uv}_{\lambda'\lambda} &= \bar{u}_\alpha(k,\lambda')\rho_{\alpha\beta}v_\beta(k,\lambda) \\ \rho^{vu}_{\lambda'\lambda} &= \bar{v}_\alpha(k,\lambda')\rho_{\alpha\beta}u_\beta(k,\lambda)\end{aligned} \tag{32}$$

The analogues of Eqs. (23) and (24) are now

$$\mathcal{M}_{+/-} =< p_\pi; X, \lambda_X \mid k, \lambda)_{+/-} = M_{+/-}(\lambda_X) e^{i\lambda\phi} d^{1/2}_{\lambda,\lambda_X}(\theta) \tag{33}$$

with

$$M_+(-\lambda_X) = \eta M_+(\lambda_X) \qquad M_-(-\lambda_X) = -\eta M_-(\lambda_X) \tag{34}$$

Consider now the dependence of the angular distribution upon the transverse polarization of the quark i.e. upon the terms in Eq. (30) involving ρ_{+-} and ρ_{-+}. For the terms containing $\rho^{uu}_{+-}, \rho^{uu}_{-+}, \rho^{vv}_{+-}$ and ρ^{vv}_{-+} the argument is identical to the free particle case and the Collins mechanism vanishes. But, for example, for the ρ^{uv}_{+-} term one has

$$\begin{aligned} \rho^{uv}_{+-} e^{i\phi} \sum_{\lambda_X} M^*_+(\lambda_X) M_-(\lambda_X) d^{1/2}_{-1/2,\lambda_X}(\theta) d^{1/2}_{1/2,\lambda_X}(\theta) = \\ \rho^{uv}_{+-} e^{i\phi} \{ M^*_+(1/2) M_-(1/2) d^{1/2}_{-1/2,1/2}(\theta) d^{1/2}_{1/2,1/2}(\theta) + \\ M^*_+(-1/2) M_-(-1/2) d^{1/2}_{-1/2,-1/2}(\theta) d^{1/2}_{1/2,-1/2}(\theta) \} \end{aligned} \tag{35}$$

$$= \rho^{uv}_{+-} e^{i\phi} M^*_+(1/2) M_-(1/2) \{ d^{1/2}_{-1/2,1/2}(\theta) d^{1/2}_{1/2,1/2}(\theta) - d^{1/2}_{-1/2,-1/2}(\theta) d^{1/2}_{1/2,-1/2}(\theta) \tag{36}$$

where we have used Eq. (34).

Note the crucial difference: the minus sign in the parenthesis in Eq. (36) compared with (28). Consequently the angular factor in Eq. (36) does *not* vanish and the Collins mechanism survives. The other contributions, from $\rho^{uv}_{-+}, \rho^{vu}_{+-}$ and ρ^{vu}_{-+} also do not vanish, and there is no cancellation between them.

Hence for the interacting, off-shell quark the Collins mechanism does not vanish, and the fundamental reason is that the operator in Eq. (14) creates a superposition of states with opposite parity, whereas in the case of a free field it creates a state with just one definite parity.

IV. DISCUSSION

We have, for simplicity, discussed the case of the semi-inclusive production of pions, but, in fact, the results hold equally well for any hadron h. Thus, in the fragmentation of a transversely polarized off-shell quark "q" $\rightarrow h + X$ where the quark is described by an interacting QCD field, the angular distribution of the h *can* depend on the direction of the quark transverse polarization i.e. the Collins mechanism is non-zero.

On the contrary, for a decaying particle, or when the quark is regarded as a "free" on-shell particle, the Collins mechanism vanishes.

The key difference between the two cases is that the operator involved in the fragmentation amplitude

$$\int d^4 z e^{-ik.z} < h; X, \lambda_X \mid \bar{\Psi}_\beta(z) \mid 0 > \qquad (k_0 > 0) \tag{37}$$

i.e. the operator

$$\bar{\Psi}_\beta(k) = \int d^4 z e^{-ik.z} \bar{\Psi}_\beta(z) \tag{38}$$

when acting on the vacuum creates a superposition of states with opposite parity when it is an interacting field,

$$\bar{\Psi}_\beta(k) \mid 0 >= \frac{1}{E_k} \sum_\lambda \{ \bar{u}_\beta(\boldsymbol{k}, \lambda) \mid k, \lambda)_+ + \bar{v}_\beta(-\boldsymbol{k}, \lambda) \mid k, \lambda)_- \} \tag{39}$$

as can be seen from Eqs. (14) to (17), whereas it creates a state with definite parity when it is a free field. The existence of a definite parity for the decaying particle is crucial for the cancellation that causes the Collins mechanism to vanish. Some attempts have been made to estimate the size of the Collins effect via dynamical calculations. For access to this literature, see [5].

It is interesting to note that there is a well known example of this parity distinction between free on-shell and interacting off-shell particles in the ancient literature [6]. In πN elastic scattering the Born term shown in Fig. (2), is known to contribute to both the s and p-wave πN states i.e.to states of opposite parity. Only when

$$s = (p_\pi + p_N)^2 \longrightarrow M^2 \tag{40}$$

where M is the nucleon mass i.e. when the exchanged nucleon is on the mass-shell, corresponding to the exchanged object being a real particle with positive parity, does the p-wave amplitude totally dominate.

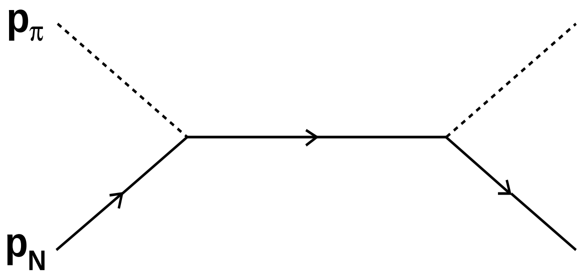

FIG. 2: Feynman diagram for πN scattering

Thus there is, in the end, no surprise that the non-vanishing of the Collins mechanism is sensitive to whether the decaying quark is treated as a "free" on-shell particle or an interacting off-shell one.

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APPENDIX: PROPERTIES OF THE STATES $\mid k, \lambda)_{+/-}$

Let $\hat{P}^\mu$ be the energy and momentum operators of the theory. Then

$$[\hat{P}^\mu, \bar{\Psi}(z)] = -i\frac{\partial \bar{\Psi}}{\partial z_\mu} \tag{A.1}$$

Substituting Eq. (11) and then integrating with $\int d^4 z e^{-ik.z}$, we obtain for the momentum operators $\hat{\boldsymbol{P}}$

$$\sum_{\lambda}\Big[\hat{\boldsymbol{P}},\{\bar{u}(\boldsymbol{k},\lambda)A^{\dagger}(\boldsymbol{k},\lambda)+\bar{v}(-\boldsymbol{k},\lambda)B(\boldsymbol{k},\lambda)\}\Big]=$$
$$\sum_{\lambda}\{\boldsymbol{k}\bar{u}(\boldsymbol{k},\lambda)A^{\dagger}(\boldsymbol{k},\lambda)+\boldsymbol{k}\bar{v}(-\boldsymbol{k},\lambda)B(\boldsymbol{k},\lambda)\} \tag{A.2}$$

Multiplying respectively by $\gamma^0 u(\boldsymbol{k},\lambda)$ and $\gamma^0 v(-\boldsymbol{k},\lambda)$ to separate the $A^{\dagger}$ and B terms we get

$$[\hat{\boldsymbol{P}},A^{\dagger}(k,\lambda)]=\boldsymbol{k}A^{\dagger}(k,\lambda) \tag{A.3}$$

$$[\hat{\boldsymbol{P}},B(k,\lambda)]=\boldsymbol{k}B(k,\lambda) \tag{A.4}$$

from which follows that the states

$$\begin{aligned} \mid k,\lambda)_{+} &= A^{\dagger}(k,\lambda)\mid 0> \\ \mid k,\lambda)_{-} &= B(k,\lambda)\mid 0> \end{aligned} \tag{A.5}$$

are eigenstates of momentum, with momentum $\boldsymbol{k}$.

To study the energy we use the fact that

$$[\hat{P}_0,a^{\dagger}(\boldsymbol{k},t)]=-i\frac{\partial a^{\dagger}(\boldsymbol{k},t)}{\partial t} \tag{A.6}$$

so that, from Eq. (15)

$$[\hat{P}_0,A^{\dagger}(k,\lambda)]=\int dte^{-ik_0t}\Big(-i\frac{\partial a^{\dagger}(\boldsymbol{k},t)}{\partial t}\Big) \tag{A.7}$$

Given that all such operator equations should really be considered within the framework of test functions, we may integrate the RHS of Eq. (A.7) by parts, discarding the terms at $t=\pm\infty$. Hence (A.7) becomes

$$[\hat{P}_0,A^{\dagger}(k,\lambda)]=k_0A^{\dagger}(k,\lambda) \tag{A.8}$$

A similar relation holds for $B(k,\lambda)$. It follows that

$$\hat{P}_0\mid k,\lambda)_{+/-}=k_0\mid k,\lambda)_{+/-} \tag{A.9}$$

Consider now the operation of space inversion

$$\mathcal{P}\bar{\Psi}(\boldsymbol{z},t)\mathcal{P}^{-1} = \bar{\Psi}(-\boldsymbol{z},t)\gamma_0 \tag{A.10}$$

It is well known that for free fields this leads to

$$\begin{aligned} \mathcal{P}a_0^\dagger(\boldsymbol{k},\lambda)\mathcal{P}^{-1} &= a_0^\dagger(-\boldsymbol{k},\lambda) \\ \mathcal{P}b_0(\boldsymbol{k},\lambda)\mathcal{P}^{-1} &= -b_0(-\boldsymbol{k},\lambda) \end{aligned} \tag{A.11}$$

On comparing Eqs. (10) and (11) it is clear, for interacting fields, that the same results will hold for $a^\dagger(\boldsymbol{k},\lambda;t)$ and $b(\boldsymbol{k},\lambda;t)$, hence that

$$\begin{aligned} \mathcal{P}A^\dagger(k,\lambda)\mathcal{P}^{-1} &= A^\dagger(\tilde{k},\lambda) \\ \mathcal{P}B(k,\lambda)\mathcal{P}^{-1} &= -B(\tilde{k},\lambda) \end{aligned} \tag{A.12}$$

where $\tilde{k} = (k_0, -\boldsymbol{k})$.

It follows that the states $\mid k,\lambda)_{+/-}$ have opposite parity.

Finally, under a Lorentz transformation l

$$U(l)\bar{\Psi}_\beta(z)U(l^{-1}) = \bar{\Psi}_\alpha(lz)D_{\alpha\beta}(l) \tag{A.13}$$

where $D_{\alpha\beta}$ corresponds to the spinor transformation matrix $S_{\alpha\beta}$ given in Bjorken and Drell [7]. If we define, as in Eq. (14),

$$\bar{\Psi}_\beta(k) = \int d^4z e^{-ik.z}\bar{\Psi}_\beta(z) \tag{A.14}$$

then

$$\begin{aligned} U(l)\bar{\Psi}_\beta(k)U(l^{-1}) &= \int d^4z e^{-ik.z}U(l)\bar{\Psi}_\beta(z)U(l^{-1}) \\ &= D_{\alpha\beta}(l)\int d^4z e^{-ik.z}\bar{\Psi}_\alpha(lz) \\ &= D_{\alpha\beta}(l)\int d^4z' e^{-ik.l^{-1}z'}\bar{\Psi}_\alpha(z') \qquad (z' = lz) \\ &= D_{\alpha\beta}(l)\int d^4z' e^{-ilk.z'}\bar{\Psi}_\alpha(z') \\ &= \bar{\Psi}_\alpha(lk)D_{\alpha\beta}(l) \end{aligned} \tag{A.15}$$

Using the expression Eq. (14) for $\bar{\Psi}_\beta(k)$ yields

$$\begin{aligned}\bar{\Psi}_\alpha(lk)D_{\alpha\beta}(l) = \frac{1}{E_{k'}}\sum_\lambda \{&\bar{u}_\alpha(\boldsymbol{k'},\lambda)A^\dagger(lk,\lambda) \\ &+\bar{v}_\alpha(-\boldsymbol{k'},\lambda)B(lk,\lambda)\}D_{\alpha\beta}(l)\end{aligned} \tag{A.16}$$

where

$$k' = lk \tag{A.17}$$

Now [7]

$$\bar{u}_\alpha(\boldsymbol{k'},\lambda)D_{\alpha\beta}(l) = \bar{u}_\beta(\boldsymbol{k},\lambda')\mathcal{D}^{1/2}_{\lambda\lambda'}(r^{-1}) \tag{A.18}$$

and

$$\bar{v}_\alpha(\boldsymbol{k'},\lambda)D_{\alpha\beta}(l) = \bar{v}_\beta(\boldsymbol{k},\lambda')\mathcal{D}^{1/2}_{\lambda'\lambda}(r) \tag{A.19}$$

where r is the Wick helicity rotation (analogous to the Wigner rotation). Thus

$$\begin{aligned}\bar{\Psi}_\alpha(lk)D_{\alpha\beta}(l) = \frac{1}{E_{k'}}\sum_\lambda \{&\bar{u}_\beta(\boldsymbol{k},\lambda')\mathcal{D}^{1/2}_{\lambda\lambda'}(r^{-1})A^\dagger(lk,\lambda) \\ &+\bar{v}_\beta(-\boldsymbol{k},\lambda')\mathcal{D}^{1/2}_{\lambda'\lambda}(r)B(lk,\lambda)\}\end{aligned} \tag{A.20}$$

Multiplying Eq. (A.13) first by $[\gamma^0 u(\boldsymbol{k},\lambda)]_\beta$ then by $[\gamma^0 v(-\boldsymbol{k},\lambda)]_\beta$, and using (A.14) and (A.20) we finally obtain

$$U(l)A^\dagger(k,\lambda)U(l^{-1}) = \frac{E_k}{E_{k'}}\mathcal{D}^{1/2}_{\lambda'\lambda}(r^{-1})A^\dagger(lk,\lambda') \tag{A.21}$$

$$U(l)B(k,\lambda)U(l^{-1}) = \frac{E_k}{E_{k'}}\mathcal{D}^{1/2}_{\lambda\lambda'}(r)B(lk,\lambda') \tag{A.22}$$

These imply that the states $\mid k,\lambda)_{+/-}$ transform just like genuine particle states under rotations, and under boosts differ from particle states only by the factor $\frac{E_k}{E_{k'}}$, which is innocuous for our purposes [8].

Thus the states $\mid k, \lambda)_{+/-}$ possess all the properties required in the derivation of the results in Sections II and III.

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[2] See for example C. Bourrely, E. Leader and J. Soffer, Phys. Rep. **59**, 95 (1980).

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[4] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, McGraw-Hill, New York (1965), Section 15.4.

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[6] S. Gasiorowicz, *Elementary Particle Physics*, John Wiley and Sons, New York (1967), Chapter 23.

[7] See, for example, Section 13.5 of [4].

[8] See Section 2.4 of [3].