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Parton intrinsic motion: suppression of the Collins mechanism for transverse single spin asymmetries in $p^\uparrow\, p \to \pi\, X$

M. Anselmino[1], M. Boglione[1], U. D'Alesio[2], E. Leader[3], F. Murgia[2]

[1]*Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy*

[2]*INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari, C.P. 170, I-09042 Monserrato (CA), Italy*

[3]*Imperial College, Prince Consort Road, London, SW7 2BW, England*

Abstract

We consider a general formalism to compute inclusive polarised and unpolarised cross sections within pQCD and the factorisation scheme, taking into account parton intrinsic motion in distribution and fragmentation functions, as well as in the elementary dynamics. Surprisingly, the intrinsic partonic motion, with all the correct azimuthal angular dependences, produces a strong suppression of the transverse single spin asymmetry arising from the Collins mechanism. As a consequence, and in contradiction with earlier claims, the Collins mechanism is unable to explain the large asymmetries found in $p^\uparrow\, p \to \pi\, X$ at moderate to large Feynman x_F. The Sivers effect is not suppressed.

1. Introduction and general formalism

The inclusive production of large p_T particles in the high energy collision of two nucleons has been for a long time a crucial testing ground for perturbative QCD; in such kinematical regions the partonic degrees of freedom dominate the hadronic processes, which can be described in terms of quark and gluon dynamics, coupled to non perturbative information – parton distribution (pdf) and fragmentation (ff) functions – gathered from other processes and evolved to the proper scale via QCD evolution equations.

In the simplest case this translates into the well known expression:

$$
\begin{aligned}
\frac{E_C \, d\sigma^{AB\to CX}}{d^3\boldsymbol{p}_C} &= \sum_{a,b,c,d} \int dx_a \, dx_b \, dz \, f_{a/A}(x_a, Q^2) \, f_{b/B}(x_b, Q^2) &(1)\\
&\times \frac{\hat{s}}{\pi z^2} \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \, \delta(\hat{s}+\hat{t}+\hat{u}) \, D_{C/c}(z, Q^2) \\
&= \sum_{a,b,c,d} \int dx_a \, dx_b \, f_{a/A}(x_a, Q^2) \, f_{b/B}(x_b, Q^2) &(2)\\
&\times \frac{1}{\pi z} \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \, D_{C/c}(z, Q^2) \,,
\end{aligned}
$$

which combines all possible elementary QCD interactions $ab \to cd$, with distribution, $f(x, Q^2)$, and fragmentation, $D(z, Q^2)$, functions: all partonic intrinsic motions have been integrated over and the hadrons are considered as composed of collinear massless quarks and gluons, each carrying a fraction x of the parent momentum; similarly for the final quark fragmentation into a collinear hadron with fraction z of the quark momentum. The energy-momentum conservation of the elementary interactions, $\hat{s}+\hat{t}+\hat{u} = 0$, allows to relate x_a, x_b and z, namely, in this collinear picture, $x_a \, x_b \, z \, s = -x_a \, t - x_b \, u$, where $\hat{s}, \hat{t}, \hat{u}$ (s, t, u) are the Mandelstam variables for the partonic (hadronic) process.

Eq. (1) – taking into account higher order contributions to the elementary interactions – describes successfully the highest energy cross section data, including the most recent ones from RHIC [1]. However, already starting from the pioneering work of Feynman, Field and Fox [2], several papers have shown that intrinsic transverse momenta $\boldsymbol{k}_\perp$'s have to be explicitly introduced into Eq. (1) in order to be able to explain data at moderately large p_T, for production of pions and photons [3, 4]; without them the theoretical (collinear) computations would give results in some cases much smaller (up to a factor 10 or even more) than experiment.

Taking into account intrinsic transverse momenta is not an entirely straightforward matter. In the pure parton model, where partons are regarded as physical particles with definite mass (usually assumed to be negligible), the standard collinear parton density $f_{a/A}(x_a)$ is simply generalised to $\hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a})$, where $\boldsymbol{k}_{\perp a}$ is the parton momentum perpendicular to the nucleon momentum, and

$$
f_{a/A}(x_a) = \int d^2\boldsymbol{k}_{\perp a} \, \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}) \,, \qquad (3)
$$

where, to be precise, x_a is the light-cone momentum fraction of parton a inside hadron A. Similarly, the fragmentation function is generalised to $\hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C})$, where $\boldsymbol{k}_{\perp C}$ is the transverse momentum of the observed hadron C with respect to the fragmenting parton c. All dynamic partonic calculations are then carried out with inclusion of the intrinsic transverse momenta $\boldsymbol{k}_\perp$'s.

This natural generalisation apparently modifies Eq. (1) into:

$$\frac{E_C \, d\sigma^{AB\to CX}}{d^3\boldsymbol{p}_C} = \tag{4}$$
$$\sum_{a,b,c,d} \int dx_a \, dx_b \, dz \, d^2\boldsymbol{k}_{\perp a} \, d^2\boldsymbol{k}_{\perp b} \, d^3\boldsymbol{k}_{\perp C} \, \delta(\boldsymbol{k}_{\perp C} \cdot \hat{\boldsymbol{p}}_c) \, \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}; Q^2) \, \hat{f}_{b/B}(x_b, \boldsymbol{k}_{\perp b}; Q^2)$$
$$\frac{\hat{s}^2}{\pi x_a x_b z^2 s} \, J(\boldsymbol{k}_{\perp C}) \, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}(\hat{s}, \hat{t}, \hat{u}, x_a, x_b) \, \delta(\hat{s} + \hat{t} + \hat{u}) \, \hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C}; Q^2) \, ,$$

where $\boldsymbol{k}_{\perp a}\,(\boldsymbol{k}_{\perp b})$ and $\boldsymbol{k}_{\perp C}$ are respectively the transverse momenta of parton $a\,(b)$ with respect to hadron $A\,(B)$, and of hadron C with respect to parton c, which in the case of light quarks or gluons is taken to be massless. We have formally extended our definition of the 2-vector $\boldsymbol{k}_{\perp C}$ into a 3-vector via the δ-function $\delta(\boldsymbol{k}_{\perp C} \cdot \hat{\boldsymbol{p}}_c)$. Neglecting parton masses, the function J is given by [4]

$$J(\boldsymbol{k}_{\perp C}) = \frac{\left(E_C + \sqrt{\boldsymbol{p}_C^2 - \boldsymbol{k}_{\perp C}^2}\right)^2}{4(\boldsymbol{p}_C^2 - \boldsymbol{k}_{\perp C}^2)} \, \cdot \tag{5}$$

Eq. (4) has been widely used in the literature, albeit without the factor J, which equals 1 if we neglect the final hadron mass and $\boldsymbol{k}_{\perp C}^2$ in Eq. (5). Note that the factor $\hat{s}/(\pi z^2)$ in Eq. (1) follows from the factor $\hat{s}^2/(\pi\, x_a x_b\, z^2 s)$ in (4) since for collinear collisions $\hat{s} = x_a x_b\, s$. Although it is true that even with $\boldsymbol{k}_\perp$, $\hat{s} \simeq x_a x_b\, s$, the use of this approximation has been shown by Cahn [5] to lead to azimuthal asymmetries which are physically impossible.

The $\boldsymbol{k}_\perp$ dependent pdf and ff are usually assumed to have simple factorised and Gaussian forms, like:

$$\hat{f}_{q/p}(x, \boldsymbol{k}_\perp; Q^2) = f_{q/p}(x, Q^2) \, g(k_\perp) = f_{q/p}(x, Q^2) \, \frac{\beta^2}{\pi} \, e^{-\beta^2 \, k_\perp^2} \, , \tag{6}$$

so that
$$\langle k_\perp^2 \rangle = 1/\beta^2 \, , \qquad \int d^2\boldsymbol{k}_\perp \, \hat{f}_{q/p}(x, \boldsymbol{k}_\perp; Q^2) = f_{q/p}(x, Q^2) \, , \tag{7}$$

where β might depend on x and the energy; it is usually assumed to be flavour independent. A similar factorisation is adopted for the $\boldsymbol{k}_\perp$ dependent fragmentation functions. The elementary cross sections $d\hat{\sigma}/d\hat{t}$ depend, via the elementary Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$, on the intrinsic motions.

The QCD factorisation theorem implicitly used in Eq. (4) – with unintegrated $\boldsymbol{k}_\perp$ dependent distribution and fragmentation functions – has never been formally proven in general [6], but only for the Drell-Yan process, for the two-particle inclusive cross section in e^+e^- annihilation [7] and, recently, for SIDIS processes in particular kinematical regions [8]. Moreover, in QCD the parton model is a *leading-twist* approximation to the theory, whereas intrinsic transverse effects are of *higher-twist* and should therefore be incorporated in a consistent higher-twist development of the theory. Unfortunately, such a treatment is very complicated and introduces a whole set of new unknown soft functions and quark-gluon correlations with unclear partonic interpretation.

It turns out, however, that some partonic effects of transverse momentum are surprisingly large and can generate phenomena which would be impossible to reproduce in the collinear treatment:

- the presence of an intrinsic $\boldsymbol{k}_\perp$ alters the relationship between the light-cone momentum fraction x of the parton and the Bjorken x_{Bj}, so that $x \neq x_{Bj}$. Although the shift is small and proportional to $\boldsymbol{k}_\perp^2/(x\sqrt{s})^2$, it can have a substantial effect in the region of x where the parton densities are varying rapidly. This is a kind of *enhanced higher-twist* effect and can lead up to an order of magnitude change in a cross section. Similarly, due to intrinsic motion, the partonic scattering angle in the pp c.m. frame might be much smaller than the hadronic production angle, thus enhancing the large p_T inclusive production of particles.

- In the presence of transverse momentum, certain spin-dependent effects can be generated by soft mechanisms and can be used to understand the large transverse single spin asymmetries (SSA) found in many reactions like $A^\uparrow + B \to C + X$ and the large hyperon polarisations in processes like $A + B \to H^\uparrow + X$. At leading twist there are 4 such soft mechanisms, often referred to as "odd under naive time reversal":
  a) *Sivers distribution function* [9]: in a transversely polarised nucleon with momentum $\boldsymbol{p}$ and polarisation vector $\boldsymbol{P}$, the number density of quarks with momentum $(x\boldsymbol{p}, \boldsymbol{k}_\perp)$ is allowed to depend upon $\boldsymbol{P} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$; in other words, the Sivers distribution function represents the azimuthal dependence (around $\boldsymbol{p}$) of the number density of unpolarised quarks inside a transversely polarised proton.
  b) *Collins fragmentation function* [6]: in the fragmentation of a transversely polarised quark with momentum $\boldsymbol{p}_q$ and polarisation vector $\boldsymbol{P}_q$, $q \to C+X$, the number density of hadrons C with momentum $(z\boldsymbol{p}_q, \boldsymbol{k}_\perp)$ is allowed to depend on $\boldsymbol{P}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{k}_\perp)$; in other words the Collins fragmentation function represents the azimuthal dependence (around $\boldsymbol{p}_q$) of the number density of unpolarised hadrons resulting from the fragmentation of a transversely polarised quark.
  c) *Boer-Mulders distribution function* [10]: in an unpolarised nucleon a quark with momentum $(x\boldsymbol{p}, \boldsymbol{k}_\perp)$ is allowed to have a non-zero polarisation along $\boldsymbol{p} \times \boldsymbol{k}_\perp$; that is, the Boer-Mulders distribution function represents the azimuthal dependence (around $\boldsymbol{p}$) of the number density of transversely polarised

quarks inside an unpolarised proton.

d) *polarising fragmentation function* [11, 12]: in the fragmentation of an unpolarised quark with momentum $\boldsymbol{p}_q$ a final spin 1/2 hadron C with momentum $(z\boldsymbol{p}_q, \boldsymbol{k}_\perp)$ is allowed to have a non-zero polarisation along $\boldsymbol{p}_q \times \boldsymbol{k}_\perp$; that is, the polarising fragmentation function represents the azimuthal dependence (around $\boldsymbol{p}_q$) of the number density of transversely polarised hadrons resulting from the fragmentation of an unpolarised quark.

It should be noted that in the pure parton model, where partons are treated as physical free particles, all these effects vanish [13].

In the present paper we study transverse single spin asymmetries, in $p^\uparrow p \to \pi X$ processes, taking into account all parton intrinsic motions in initial and final hadrons and in the elementary dynamics. This generalises previous work in which only the one $\boldsymbol{k}_\perp$ essential to the mechanism was taken into account, either in the initial polarised nucleon (Sivers effect) or in the final quark fragmentation (Collins effect), and the $\boldsymbol{k}_\perp$ distribution was somewhat simplified into essentially a two-dimensional δ-function [14, 15, 16].

For the reasons explained above, we have not attempted to construct a fully consistent next-to-leading-twist treatment. Our strategy is to keep only the enhanced higher-twist terms and to calculate partonic helicity amplitudes as if the partons were particles. We believe this approach is physically meaningful since it takes into account the most important higher-twist terms in the cross section and the asymmetry. Three of the above spin effects, a)–c), can contribute to pion SSA, but in this paper we wish to explore the generation of SSA due to the existence of the Collins fragmentation function alone, for which there is some evidence in the polarised lepto-production data of the HERMES collaboration [17, 18]. The Boer-Mulders effect can also contribute to transverse single spin asymmetries but, at least for $p^\uparrow p \to \pi X$ processes, it would contribute mainly at negative x_F values, whereas data are in the positive x_F region. The Sivers effect is also relevant, and has been studied in a parallel paper [4]. In fact we shall show that the consistent treatment of all intrinsic partonic motions induces a major suppression of the contribution to the asymmetry due to the Collins mechanism and renders it incapable of producing, by itself, the kind of asymmetries measured in $p^\uparrow p \to \pi X$ reactions [19, 20].

This result modifies the conclusions of Ref. [15, 16], where the Collins contribution to SSA in $p^\uparrow p \to \pi X$ processes was computed adopting a simplified kinematical configuration: it appeared that the Collins fragmentation function could, although with some difficulty, explain the E704 data [19]. Note that the results on SSA obtained using the Sivers distribution function, with a similar simplified kinematical configuration [14] are, instead, essentially confirmed by the exact treatment of all intrinsic partonic motions [4].

In order to study spin asymmetries we have to introduce spins in the QCD hard scattering processes. Eq. (1) holds also for polarised processes, $(A, S_A) + (B, S_B) \to C + X$ [21], provided one introduces in the factorisation scheme, in addition to the distribution functions, the helicity density matrices which describe the parton spin states. This can be done also for Eq. (4) with the result:

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)$$
$$J(\boldsymbol{k}_{\perp C})\, \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \tag{8}$$
$$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,$$

where we have used the notation $\{\lambda\}$ to imply a sum over *all* helicity indices. In Eq. (8) $\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}$ is the helicity density matrix of parton a inside the polarised hadron A whose polarisation state is generically labelled by S_A (for spin 1/2 particles this means longitudinal or transverse polarisation); similarly for parton b inside hadron B with spin S_B. The $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$'s are the helicity amplitudes for the elementary process $ab\to cd$, normalised so that the unpolarised cross section, for a collinear collision, is given by

$$\frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}\frac{1}{4}\sum_{\lambda_a,\lambda_b,\lambda_c,\lambda_d} |\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}|^2\,. \tag{9}$$

$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$ is the product of *fragmentation amplitudes* for the $c\to C+X$ process

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c} = \not\!\!\!\!\sum\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_X,\lambda_C;\lambda_c}\, \hat{\mathcal{D}}^*_{\lambda_X,\lambda'_C;\lambda'_c}\,, \tag{10}$$

where the $\not\!\!\!\!\sum\!\!\!\!\int_{X,\lambda_X}$ stands for a spin sum and phase space integration over all undetected particles, considered as a system X. The usual unpolarised fragmentation function $D_{C/c}(z)$, *i.e.* the number density of hadrons C resulting from the fragmentation of an unpolarised parton c and carrying a light-cone momentum fraction z, is given by

$$D_{C/c}(z) = \frac{1}{2}\sum_{\lambda_c,\lambda_C}\int d^2\boldsymbol{k}_{\perp C}\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda_c}(z,\boldsymbol{k}_{\perp C})\,. \tag{11}$$

Eq. (8) can be formally simplified, showing its physical meaning, by noticing that:

$$\sum_{\lambda_a,\lambda_b,\lambda'_a,\lambda'_b,\lambda_d} \rho^{A,S_A}_{\lambda_a,\lambda'_a}\, \rho^{B,S_B}_{\lambda_b,\lambda'_b}\, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} = \rho'_{\lambda_c,\lambda'_c}(c) = \rho_{\lambda_c,\lambda'_c}\, \mathrm{Tr}\rho'(c)\,, \tag{12}$$

where $\rho_{\lambda_c,\lambda'_c}$ is the normalised helicity density matrix of parton c produced in the $ab\to cd$ process, with initially polarised partons a and b; the normalisation factor $\mathrm{Tr}\rho'(c)$ is related to the polarised cross section for a collinear collision:

$$\mathrm{Tr}\rho'(c) = (32\pi^2\hat{s}^2)\, \frac{d^2\hat{\sigma}^{(a,s_a)+(b,s_b)\to c+d}}{d\hat{t}\, d\hat{\phi}}\,, \tag{13}$$

where $\hat{\phi}$ is the azimuthal angle of parton c in the partonic center of mass frame. Moreover,

$$\sum_{\lambda_c \lambda'_c, \lambda_C} \rho_{\lambda_c, \lambda'_c} \, \hat{D}^{\lambda_C, \lambda_C}_{\lambda_c, \lambda'_c}(z, \boldsymbol{k}_{\perp C}) = \hat{D}_{C/c, s_c}(z, \boldsymbol{k}_{\perp C}) \,, \tag{14}$$

is just the fragmentation function of a polarised parton c, with spin configuration s_c, into a hadron C, whose spin is not observed.

Using Eqs. (12)-(14), Eq. (8) can be written as:

$$\frac{E_C \, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d} \int dx_a \, dx_b \, dz \, d^2\boldsymbol{k}_{\perp a} \, d^2\boldsymbol{k}_{\perp b} \, d^3\boldsymbol{k}_{\perp C} \, \delta(\boldsymbol{k}_{\perp C} \cdot \hat{\boldsymbol{p}}_c)$$
$$\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) \, \hat{f}_{b/B,S_B}(x_b, \boldsymbol{k}_{\perp b}) \, \frac{2\hat{s}^2}{x_a x_b z^2 s} \, J(\boldsymbol{k}_{\perp C})$$
$$\frac{d^2\hat{\sigma}^{(a,s_a)+(b,s_b)\to c+d}}{d\hat{t} \, d\hat{\phi}} \, \delta(\hat{s}+\hat{t}+\hat{u}) \, \hat{D}_{C/c,s_c}(z, \boldsymbol{k}_{\perp C}) \,, \tag{15}$$

which is the analogue of Eq. (4) in the polarised case.

Eq. (15) shows clearly the factorised structure and the partonic interpretation: inside polarised hadrons one has polarised partons with spin configurations s_a and s_b, which interact via pQCD processes, leading to a final polarised parton, with spin configuration s_c, which fragments into the observed final hadron. For the initial and final step – the determination of the parton polarisation from the hadron polarisation and the fragmentation of the polarised parton – one has to rely on distribution and fragmentation functions; some of them are known from other processes or from theoretical models and some of them, in particular when allowing for intrinsic motions, are new and unexplored.

Although Eq. (15) has a simple physical interpretation, it is more convenient to study the scattering process with the helicity formalism of Eq. (8); when dealing with helicities and helicity density matrices all spins have a well defined interpretation concerning their directions [22], and this is crucial if we are taking into account all parton transverse motions, so that there are several transverse spin directions. Since the direction of motion of the parton does not coincide with that of its parent hadron, the longitudinal and transverse direction of the parton spin will also be different from the longitudinal and transverse direction of the parent hadron spin.

The partonic distribution is usually regarded, at Leading Order, as the inclusive cross section for the process $A \to a + X$; therefore the helicity density matrix of parton a inside a hadron A with polarisation S_A can be written as

$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a} \, \hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) = \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \, \mathop{\not\!\!\int\!\!\!\!\!\!\sum}_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A} \, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A} \tag{16}$$
$$= \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \,, \tag{17}$$

having defined

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \equiv \nfint_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A} \, \hat{\mathcal{F}}^*_{\lambda'_a,\lambda_{X_A};\lambda'_A} \,, \tag{18}$$

and where the $\nfint_{X_A,\lambda_{X_A}}$ stands for a spin sum and phase space integration over all undetected remnants of hadron A, considered as a system X_A and the $\hat{\mathcal{F}}$'s are the *helicity distribution amplitudes* for the $A \to a + X$ process.

Notice that Eq. (17) relates the helicity density matrix of parton a to the helicity density matrix of hadron A. The helicity density matrix describes the spin orientation of a particle in its *helicity rest frame* [22]; for a spin 1/2 particle, $\mathrm{Tr}\,(\sigma_i \rho) = P_i$ is the i-component of the polarisation vector $\boldsymbol{P}$ in the helicity rest frame of the particle. In this sense Eq. (17) relates the hadron polarisation to the parton polarisation, which have both to be defined and interpreted in the proper rest frames.

The distribution function of parton a inside the polarised hadron A, S_A is given by

$$\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a}) = \sum_{\lambda_a,\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \, \hat{F}^{\lambda_a,\lambda_a}_{\lambda_A,\lambda'_A} \tag{19}$$

and the usual unpolarised distribution function $f_{a/A}(x_a)$, *i.e.* the number density of partons a inside an unpolarised parton A, carrying a light-cone momentum fraction x_a, is given by

$$f_{a/A}(x_a) = \frac{1}{2} \sum_{\lambda_a,\lambda_A} \int d^2\boldsymbol{k}_{\perp a} \, \hat{F}^{\lambda_a,\lambda_a}_{\lambda_A,\lambda_A} \,. \tag{20}$$

Similar expressions for the fragmentation process have already been introduced in Eqs. (10) and (11).

By using Eq. (17), Eq. (8) can be written as

$$\frac{E_C \, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a \, dx_b \, dz}{16\pi^2 x_a x_b z^2 s} \, d^2\boldsymbol{k}_{\perp a} \, d^2\boldsymbol{k}_{\perp b} \, d^3\boldsymbol{k}_{\perp C} \, \delta(\boldsymbol{k}_{\perp C} \cdot \hat{\boldsymbol{p}}_c)$$

$$J(\boldsymbol{k}_{\perp C}) \, \rho^{A,S_A}_{\lambda_A,\lambda'_A} \, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A} \, \rho^{B,S_B}_{\lambda_B,\lambda'_B} \, \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda'_B} \, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b} \, \delta(\hat{s}+\hat{t}+\hat{u}) \, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c} . \tag{21}$$

Eq. (21) contains all possible combinations of different distribution and fragmentation amplitudes: these combinations have partonic interpretations and are related to the $\boldsymbol{k}_\perp$ and spin dependent fragmentation and distribution functions discussed above and, for example, in Refs. [23] and [24]. Notice that, even though Eq. (4), for the unpolarised cross section, looks intuitively correct and convincing, Eq. (21), if Collins and Boer-Mulders effects are operative, will yield a different result, *i.e.* with $\rho^I_{\lambda_I,\lambda'_I} = (1/2)\,\delta_{\lambda_I,\lambda'_I}$ ($I = A, B$), Eq. (21) contains terms not included in Eq. (4), that is the terms off-diagonal in the parton helicities. We have checked numerically that these contributions are negligible in the unpolarised cross section. All this will be discussed in detail in a forthcoming paper [25], where all contributions to single and double spin asymmetries will be examined, together with the parity and $\boldsymbol{k}_\perp$

properties of the distribution and fragmentation amplitudes. Here we are only considering the process $p^\uparrow p \to \pi X$ and are focussing only on the contribution of Collins mechanism [6], that is the azimuthal dependence of the number of pions created in the fragmentation of a transversely polarised quark. The unpolarised cross section will be computed according to Eq. (4), taking into account the intrinsic transverse motion of all partons (see also Ref. [4]).

2. Single Spin Asymmetries and Collins mechanism for pion production

Let us then consider the processes $p^\uparrow(p^\downarrow)\, p \to \pi X$; we study them in the pp center of mass frame, with the polarised beam moving along the positive Z-axis and the pion produced in the XZ plane with $(\boldsymbol{p}_\pi)_x > 0$ values. The $\uparrow$ ($\downarrow$) is defined as the $+Y(-Y)$ direction. We then have, with $S_A = \uparrow, \downarrow$, and with an unpolarised hadron B ($S_B = 0$),

$$\rho^{A,\uparrow\downarrow}_{\lambda_A,\lambda'_A} = \frac{1}{2}\begin{pmatrix} 1 & \mp i \\ \pm i & 1 \end{pmatrix} \qquad \rho^{B,0}_{\lambda_B,\lambda'_B} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{22}$$

The computation of the single spin asymmetry

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \tag{23}$$

requires evaluation and integration, for each elementary process $ab \to cd$, of the quantity [see Eq. (21)]

$$\Sigma(S_A, S_B) \equiv \sum_{\{\lambda\}} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\, \hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}\, \rho^{B,S_B}_{\lambda_B,\lambda'_B}\, \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda'_B}\, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \hat{D}^\pi_{\lambda_c,\lambda'_c}\,, \tag{24}$$

where $\hat{D}^\pi_{\lambda_c,\lambda'_c}$ is defined as in Eq. (10), for pion production. From Eqs. (22) and (24) one has that the numerator of A_N is proportional to

$$\Sigma(\uparrow,0) - \Sigma(\downarrow,0) = \sum_{\{\lambda\}} \frac{(-i)}{2}\left[\hat{F}^{\lambda_a,\lambda'_a}_{+,-} - \hat{F}^{\lambda_a,\lambda'_a}_{-,+}\right] \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda_B}\, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \hat{D}^\pi_{\lambda_c,\lambda'_c}\,, \tag{25}$$

while the denominator contains:

$$\Sigma(\uparrow,0)+\Sigma(\downarrow,0) = \sum_{\{\lambda\}} \frac{1}{2}\left[\hat{F}^{\lambda_a,\lambda'_a}_{+,+} + \hat{F}^{\lambda_a,\lambda'_a}_{-,-}\right] \hat{F}^{\lambda_b,\lambda'_b}_{\lambda_B,\lambda_B}\, \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \hat{D}^\pi_{\lambda_c,\lambda'_c}. \tag{26}$$

In the equations above and in the sequel $+$ and $-$ stand for $+1/2$ and $-1/2$ helicities, when referring to nucleons or quarks, and for $+1$ and -1 helicities, when referring to gluons.

As we have said, in this paper we are focussing solely on the Collins mechanism and we do not consider all possible contributions to A_N, which will be discussed elsewhere [25]. Therefore, we do not consider the possibility of finding transversely polarised quarks inside the unpolarised proton B [10] or the possibility of having different total numbers of quarks, at different $\boldsymbol{k}_\perp$ values, inside the transversely polarised proton A [9]. This does not imply that these other effects which are negligible for the unpolarised cross section are negligible for the SSA; simply that we wish to explore to what extent the Collins mechanism alone is able to explain the measured transverse single spin asymmetries. As a consequence, the $\hat{F}$-terms off-diagonal in λ_b, λ'_b (while diagonal in λ_B, λ'_B) and the $\hat{F}$-terms off-diagonal in λ_A, λ'_A (while diagonal in λ_a, λ'_a) will be neglected. The Collins mechanism corresponds to the terms off-diagonal in the fragmenting quark helicities λ_c, λ'_c. Taking all this into account, a partial summation in Eq. (25) obtains

$$\begin{aligned}
&\Sigma(\uparrow, 0) - \Sigma(\downarrow, 0) = \sum_{\{\lambda\}} \frac{(-i)}{2} \Big\{ \\
&\left[\hat{F}^{+,-}_{+,-} - \hat{F}^{+,-}_{-,+}\right]_{a/A} \hat{f}_{b/B}\, \hat{M}_{\lambda_c,\lambda_d;+,\lambda_b}\, \hat{M}^*_{-\lambda_c,\lambda_d;-,\lambda_b}\, \hat{D}^\pi_{\lambda_c,-\lambda_c} + \\
&\left[\hat{F}^{-,+}_{+,-} - \hat{F}^{-,+}_{-,+}\right]_{a/A} \hat{f}_{b/B}\, \hat{M}_{\lambda_c,\lambda_d;-,\lambda_b}\, \hat{M}^*_{-\lambda_c,\lambda_d;+,\lambda_b}\, \hat{D}^\pi_{\lambda_c,-\lambda_c} \Big\} ,
\end{aligned} \tag{27}$$

where we have exploited the fact that, by parity invariance,

$$\hat{F}^{\lambda_b,\lambda_b}_{+,+} + \hat{F}^{\lambda_b,\lambda_b}_{-,-} = \hat{f}_{b/B} , \tag{28}$$

independently of the value of λ_b.

The same procedure applied to Eq. (26) reveals that the denominator of A_N is just twice the unpolarised cross section, as given by Eq. (4).

Eq. (27) can be further simplified by exploiting the dynamical and the parity properties of the helicity amplitudes appearing in it. This requires some careful considerations.

- Whereas the hadronic process $p^\uparrow p \to \pi X$ takes place, according to our choice, in the XZ plane, all other elementary processes involved: $A(B) \to a(b) + X$, $ab \to cd$ and $c \to \pi + X$, do not; all parton and hadron momenta, $\boldsymbol{p}_a$, $\boldsymbol{p}_b$, $\boldsymbol{p}_C$, have transverse components $\boldsymbol{k}_{\perp a}$, $\boldsymbol{k}_{\perp b}$, $\boldsymbol{k}_{\perp C}$ and this complicates remarkably the kinematics. For example, the elementary QCD process $ab \to cd$, whose helicity amplitudes are well known in the ab center of mass frame, is not, in general, a planar process anymore when observed from the pp center of mass frame. Similarly, as we commented, the spin properties described by helicity density matrices have clear physical interpretations in each particle's own helicity rest frame, but not necessarily in the pp center of mass frame. Of course, one can always boost and rotate from one frame into another, but this introduces phases in the helicity amplitudes, which have to be properly accounted for.

- We refer all angles to the pp c.m. frame, in which $\hat{\boldsymbol{p}}_i = (\theta_i, \phi_i)$, $(i = a, b, c, d)$. Then the distribution functions of the polarised proton A describe processes taking place in the plane defined by Z and the $\boldsymbol{p}_a$ direction, (θ_a, ϕ_a). Therefore [22, 25]
$$\hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a, \boldsymbol{k}_{\perp a}) = \mathcal{F}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a, k_{\perp a}) \exp[i\lambda_A \phi_a] \tag{29}$$
and
$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, \boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, k_{\perp a}) \exp[i(\lambda_A - \lambda'_A)\phi_a]\,, \tag{30}$$
where $k_{\perp a} = |\boldsymbol{k}_{\perp a}|$; $F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, k_{\perp a})$ has the same definition as $\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a, \boldsymbol{k}_{\perp a})$, Eq. (18), with $\hat{\mathcal{F}}$ replaced by $\mathcal{F}$.
The parity properties of $\mathcal{F}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a, k_{\perp a})$ are the usual ones valid for helicity amplitudes in the $\phi_a = 0$ plane [22],
$$\mathcal{F}_{-\lambda_a,-\lambda_{X_A};-\lambda_A} = \eta\,(-1)^{S_A - s_a - S_{X_A}}\,(-1)^{\lambda_A - \lambda_a + \lambda_{X_A}}\,\mathcal{F}_{\lambda_a,\lambda_{X_A};\lambda_A}\,, \tag{31}$$
where η is an intrinsic parity factor such that $\eta^2 = 1$. These imply:
$$F^{-\lambda_a,-\lambda'_a}_{-\lambda_A,-\lambda'_A} = (-1)^{2(S_A - s_a)}\,(-1)^{(\lambda_A - \lambda_a) + (\lambda'_A - \lambda'_a)}\,F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}\,. \tag{32}$$

- Let us consider now the elementary partonic amplitudes. As already remarked, the hard partonic interactions, $a(\boldsymbol{p}_a) + b(\boldsymbol{p}_b) \to c(\boldsymbol{p}_c) + d(\boldsymbol{p}_d)$, take place out of the XZ plane, which we have chosen as the plane of the overall $p^\uparrow p \to \pi X$ process. One could compute the helicity amplitudes for these generic processes among massless particles using techniques well known in the literature, like those explained in Chapter 10 of Ref. [22]. On the other hand, the explicit expressions and the parity properties of the helicity amplitudes $\hat{M}^0$, which apply when the elementary scatterings occur in the ab c.m. frame, in the XZ plane, are well known. Therefore, rather than computing directly the generic helicity amplitudes $\hat{M}$, we prefer to relate them to the known amplitudes $\hat{M}^0$.
To reach the simple configuration of the $\hat{M}^0$ amplitudes, starting from the generic configuration $\boldsymbol{p}_a$, $\boldsymbol{p}_b$, we have to perform a boost in the direction determined by $(\boldsymbol{p}_a + \boldsymbol{p}_b)$ so that the boosted three-vector $(\boldsymbol{p}'_a + \boldsymbol{p}'_b)$ is equal to zero. This will provide us with a c.m.-like reference frame S' where the partons a and b collide head-on. Here the parton a and the parton c, resulting from the hard interaction between a and b, will have directions identified by (θ'_a, ϕ'_a) and (θ'_c, ϕ'_c) respectively. In general, the parton momenta in S' are related to the initial ones (before the boost) by:
$$\boldsymbol{p}'_i = \boldsymbol{p}_i - \frac{\boldsymbol{q}}{q^0 + \sqrt{q^2}}\left(\frac{p_i \cdot q}{\sqrt{q^2}} + p_i^0\right) \tag{33}$$
where $i = a, b, c, d$ and $q^\mu = (q^0, \boldsymbol{q}) = p_a^\mu + p_b^\mu$.

We need now to perform two subsequent rotations, one around the Z axis by an angle ϕ'_a, and one around the Y axis, by an angle θ'_a, such that the collision axis of the two colliding initial partons turns out to be aligned with the Z axis. We call this frame S''.

Under these boost and rotations the helicity states and consequently the scattering amplitudes acquire phases, $\xi_{a,b,c,d}$ and $\tilde{\xi}_{a,b,c,d}$:

$$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \hat{M}^{S''}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, e^{-i(\lambda_a\xi_a+\lambda_b\xi_b-\lambda_c\xi_c-\lambda_d\xi_d)}\, e^{-i[(\lambda_a-\lambda_b)\tilde{\xi}_a-(\lambda_c-\lambda_d)\tilde{\xi}_c]}\,, \quad (34)$$

where ξ_j and $\tilde{\xi}_j$ $(j = a, b, c, d)$ are defined by [22]

$$\cos\xi_j = \frac{\cos\theta_q \sin\theta_j - \sin\theta_q \cos\theta_j \cos(\phi_q - \phi_j)}{\sin\theta_{qp_j}} \quad (35)$$

$$\sin\xi_j = \frac{\sin\theta_q \sin(\phi_q - \phi_j)}{\sin\theta_{qp_j}}\,; \quad (36)$$

and

$$\tilde{\xi}_j = \eta'_j + \xi'_j\,, \quad (37)$$

where

$$\cos\xi'_j = \frac{\cos\theta_q \sin\theta'_j - \sin\theta_q \cos\theta'_j \cos(\phi_q - \phi'_j)}{\sin\theta_{qp'_j}} \quad (38)$$

$$\sin\xi'_j = \frac{-\sin\theta_q \sin(\phi_q - \phi'_j)}{\sin\theta_{qp'_j}}\,; \quad (39)$$

$$\cos\eta'_j = \frac{\cos\theta'_a - \cos\theta'_j \cos\theta_{p'_ap'_j}}{\sin\theta'_j \sin\theta_{p'_ap'_j}} \quad (40)$$

$$\sin\eta'_j = \frac{\sin\theta'_a \sin(\phi'_a - \phi'_j)}{\sin\theta_{p'_ap'_j}}\,, \quad (41)$$

and the polar angles (θ'_j, ϕ'_j) are determined via Eq. (33). Here $\theta_{p_ip_j}$ $(0 \le \theta_{p_ip_j} \le \pi)$ is the angle between $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$, and so on. Notice that $\eta'_a = 0$.

In the S'' frame the direction of the parton c is characterised by an azimuthal angle ϕ''_c given by

$$\tan\phi''_c = \frac{\sin\theta'_c \sin(\phi'_c - \phi'_a)}{\sin\theta'_c \cos(\phi'_c - \phi'_a)\cos\theta'_a - \cos\theta'_c \sin\theta'_a}\,. \quad (42)$$

A final rotation around Z of an angle ϕ''_c will then finally bring us to the *canonical* configuration in which the partonic process is a c.m. one in the XZ plane. This introduces another phase. As a result of the performed boost and rotations the elementary scattering amplitudes computed in the hadronic c.m. system (the one where we are studying the hadronic cross section) are related to the helicity amplitudes computed in the partonic c.m. system (in the XZ plane, $\phi_c = 0$) by:

$$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, e^{-i(\lambda_a\xi_a+\lambda_b\xi_b-\lambda_c\xi_c-\lambda_d\xi_d)} \, e^{-i[(\lambda_a-\lambda_b)\tilde{\xi}_a-(\lambda_c-\lambda_d)\tilde{\xi}_c]} \, e^{i(\lambda_a-\lambda_b)\phi''_c} \tag{43}$$

with ϕ''_c, ξ_j and $\tilde{\xi}_j$ defined in Eqs. (35)–(42); Eq. (33) allows to fully express the amplitudes in terms of the pp c.m. variables $\boldsymbol{p}_i$. The parity properties of the canonical c.m. amplitudes $\hat{M}^0$ are the usual ones:

$$\hat{M}^0_{-\lambda_c,-\lambda_d;-\lambda_a,-\lambda_b} = \eta_a\eta_b\eta_c\eta_d(-1)^{s_a+s_b-s_c-s_d} \, (-1)^{(\lambda_a-\lambda_b)-(\lambda_c-\lambda_d)} \hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} \, , \tag{44}$$

where η_i is the intrinsic parity factor for particle i.

- Let us finally consider the fragmentation process. We take as independent variables, in the pp c.m. frame, the four-momentum of the final hadron $p^\mu_C \equiv p^\mu_\pi = (\sqrt{p_T^2+p_L^2}, p_T, 0, p_L)$ (whose three-momentum, according to our choice, lies in the hadronic XZ plane and where we neglect the pion mass), the intrinsic transverse momentum $\boldsymbol{k}_{\perp C} \equiv \boldsymbol{k}_{\perp\pi} = (k_{\perp\pi}, \theta_{k_{\perp\pi}}, \phi_{k_{\perp\pi}})$ of the final pion with respect to $\boldsymbol{p}_c$ ($\boldsymbol{k}_{\perp\pi}\cdot\boldsymbol{p}_c = 0$), and the light-cone momentum fraction $z = p^+_\pi/p^+_c$.

  The parity properties of the fragmentation amplitudes, Eq. (10), are simple – analogous to the ones for the distribution amplitudes, Eqs. (31) and (32) – in a frame S^H in which the parton c moves along the Z^H-axis. This frame can be reached from the hadronic pp frame by performing two rotations: first around Z by an angle ϕ_c and then around the new Y-axis by an angle θ_c, which brings the 3-momentum $\boldsymbol{p}_c$ of parton c along the new Z^H-axis. In the frame S^H the azimuthal angle ϕ^H_π identifying the direction of the final detected pion (which coincides with the azimuthal angle of $\boldsymbol{k}_{\perp\pi}$ in S^H) is given, in terms of our chosen pp c.m. variables, by

$$\tan\phi^H_\pi = \pm\frac{p_T}{\sqrt{E_\pi^2-k_{\perp\pi}^2}}\sqrt{1-\left(\frac{k_{\perp\pi}-p_L\cos\theta_{k_{\perp\pi}}}{p_T\sin\theta_{k_{\perp\pi}}}\right)^2}\,\tan\theta_{k_{\perp\pi}} \, , \tag{45}$$

  where $E_\pi = \sqrt{p_T^2+p_L^2}$ is the energy of the final pion.

  The analogue of Eqs. (29), (30) and (32), for the fragmentation of a parton c into a pion, reads

$$\hat{\mathcal{D}}_{\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp\pi}) = \mathcal{D}_{\lambda_X;\lambda_c}(z,k_{\perp\pi})\exp[i\lambda_c\phi^H_\pi] \, , \tag{46}$$

$$\hat{D}^\pi_{\lambda_c,\lambda'_c} = D^\pi_{\lambda_c,\lambda'_c}\exp[i(\lambda_c-\lambda'_c)\phi^H_\pi] \, , \tag{47}$$

with the parity relationships

$$D^{\pi}_{-\lambda_c,-\lambda'_c} = (-1)^{2s_c}\,(-1)^{\lambda_c+\lambda'_c}\,D^{\pi}_{\lambda_c,\lambda'_c}\,, \tag{48}$$

where $D^{\pi}_{\lambda_c,\lambda'_c}$ is defined according to Eq. (10), in the case in which the hadron C is a spinless particle (pion),

$$D^{\pi}_{\lambda_c,\lambda'_c}(z,k_{\perp\pi}) = \sum\!\!\!\!\!\!\!\int_{X,\lambda_X} \mathcal{D}_{\lambda_X;\lambda_c}\,\mathcal{D}^*_{\lambda_X;\lambda'_c}\,. \tag{49}$$

By exploiting the above angular and parity relations, Eqs. (30), (32), (43), (44), (47) and (48), we can now further simplify Eq. (27). One obtains:

$$\begin{aligned}\Sigma(\uparrow,0)-\Sigma(\downarrow,0) &= -i\sum_{\{\lambda\}}\left[\hat f_{b/B}\,\hat M^0_{\lambda_c,\lambda_d;+,\lambda_b}\,\hat M^{0\,*}_{-\lambda_c,\lambda_d;-,\lambda_b}\,D^{\pi}_{\lambda_c,-\lambda_c}\right]\\ &\quad\Big\{F^{+-}_{+-}\,\cos[\phi_a+\phi''_c-\xi_a-\tilde\xi_a+2\,\lambda_c(\xi_c+\tilde\xi_c+\phi^H_\pi)]\\ &\quad-\,F^{+-}_{-+}\,\cos[\phi_a-\phi''_c+\xi_a+\tilde\xi_a-2\,\lambda_c(\xi_c+\tilde\xi_c+\phi^H_\pi)]\Big\}\,,\end{aligned} \tag{50}$$

where we have also used the fact that partons a and c, carrying transverse polarisation, are quarks or antiquarks, that is $s_a = s_c = 1/2$.

Let us finally perform the remaining sum over helicities in Eq. (50). The only types of elementary interactions contributing are $q_aq_b \to q_cq_d$ (generically denoted as qq) and $qg \to qg$ (generically denoted as qg), where $q_a = u, d, s, \bar u, \bar d, \bar s$ and so on. The only independent helicity amplitudes $\hat M^0$ for the qq processes are:

$$\begin{aligned}\hat M^0_{+,+;+,+} &= \hat M^0_{-,-;-,-} \equiv (\hat M^0_1)_{qq}\\ \hat M^0_{-,+;-,+} &= \hat M^0_{+,-;+,-} \equiv (\hat M^0_2)_{qq}\\ \hat M^0_{-,+;+,-} &= \hat M^0_{+,-;-,+} \equiv (\hat M^0_3)_{qq}\,.\end{aligned} \tag{51}$$

and, for the qg processes,

$$\hat M^0_{+,1;+,1} = \hat M^0_{-,-1;-,-1} \equiv (\hat M^0_1)_{qg} \qquad \hat M^0_{-,1;-,1} = \hat M^0_{+,-1;+,-1} \equiv (\hat M^0_2)_{qg}\,. \tag{52}$$

At Leading Order all such amplitudes are real.

On summing over $\{\lambda\}$ Eq. (50) gives, for qq processes,

$$\begin{aligned}[\Sigma(\uparrow,0)-\Sigma(\downarrow,0)]_{qq} &= \Big\{F^{+-}_{+-}(x_a,k_{\perp a})\,\cos[\phi_a+\phi''_c-\xi_a-\tilde\xi_a+\xi_c+\tilde\xi_c+\phi^H_\pi]\\ &\quad-\,F^{+-}_{-+}(x_a,k_{\perp a})\,\cos[\phi_a-\phi''_c+\xi_a+\tilde\xi_a-\xi_c-\tilde\xi_c-\phi^H_\pi]\Big\}\\ &\quad\times\hat f_{q/B}(x_b,k_{\perp b})\,\left[\hat M^0_1\hat M^0_2(x_a,x_b,z;\boldsymbol{k}_{\perp a},\boldsymbol{k}_{\perp b},\boldsymbol{k}_{\perp\pi})\right]_{qq}\,\left[-2iD^{\pi}_{+-}(z,k_{\perp\pi})\right]\end{aligned} \tag{53}$$

and, for qg processes

$$\begin{aligned}[\Sigma(\uparrow,0)-\Sigma(\downarrow,0)]_{qg} &= \Big\{F^{+-}_{+-}(x_a,k_{\perp a})\,\cos[\phi_a+\phi''_c-\xi_a-\tilde\xi_a+\xi_c+\tilde\xi_c+\phi^H_\pi]\\ &\quad-\,F^{+-}_{-+}(x_a,k_{\perp a})\,\cos[\phi_a-\phi''_c+\xi_a+\tilde\xi_a-\xi_c-\tilde\xi_c-\phi^H_\pi]\Big\}\\ &\quad\times\hat f_{g/B}(x_b,k_{\perp b})\,\left[\hat M^0_1\hat M^0_2(x_a,x_b,z;\boldsymbol{k}_{\perp a},\boldsymbol{k}_{\perp b},\boldsymbol{k}_{\perp\pi})\right]_{qg}\,\left[-2iD^{\pi}_{+-}(z,k_{\perp\pi})\right]\,.\end{aligned} \tag{54}$$

The product of amplitudes appearing in Eqs. (53) and (54) are given by:

$$\begin{aligned}
\hat{M}_1^0\,\hat{M}_2^0 &= g_s^4\,\frac{8}{9}\left\{-\frac{\hat{s}\hat{u}}{\hat{t}^2}+\delta_{\alpha\beta}\,\frac{1}{3}\,\frac{\hat{s}}{\hat{t}}\right\} && (q_\alpha q_\beta\to q_\alpha q_\beta)\\
\hat{M}_1^0\,\hat{M}_2^0 &= g_s^4\,\frac{8}{9}\,\delta_{\alpha\gamma}\left\{-\frac{\hat{s}\hat{u}}{\hat{t}^2}+\delta_{\alpha\beta}\,\frac{1}{3}\,\frac{\hat{u}}{\hat{t}}\right\} && (q_\alpha\bar{q}_\beta\to q_\gamma\bar{q}_\delta) \qquad (55)\\
\hat{M}_1^0\,\hat{M}_2^0 &= g_s^4\,\frac{8}{9}\left\{\frac{9}{4}\,\frac{\hat{s}\hat{u}}{\hat{t}^2}-1\right\} && (qg\to qg)
\end{aligned}$$

where α,β,γ and δ are flavour indices. Notice that in the above expressions all the dependences on the angles in the distribution and fragmentation functions are explicit and the functions F, $\hat{f}$ and D do not depend on angles any more; the elementary amplitudes depend on angles via the Mandelstam variables $\hat{s},\hat{t}$ and $\hat{u}$. Notice also that the qq and qg contributions have exactly the same structure, the difference being only in the parton b distribution and in the elementary processes.

From Eqs. (21), (24), (53) and (54) the numerator of the single spin asymmetry A_N, under the assumption that only Collins effect contributes, is given by (b,d can be either quarks or gluons):

$$\begin{aligned}
\frac{E_\pi\,d\sigma^{p^\uparrow p\to\pi X}}{d^3\boldsymbol{p}_\pi} &- \frac{E_\pi\,d\sigma^{p^\downarrow p\to\pi X}}{d^3\boldsymbol{p}_\pi} = \qquad (56)\\
&\sum_{q_a,b,q_c,d}\int\frac{dx_a\,dx_b\,dz}{16\pi^2 x_a x_b z^2 s}\;d^2\boldsymbol{k}_{\perp a}\,d^2\boldsymbol{k}_{\perp b}\,d^3\boldsymbol{k}_{\perp\pi}\,\delta(\boldsymbol{k}_{\perp\pi}\cdot\hat{\boldsymbol{p}}_c)\\
&\times\;J(\boldsymbol{k}_{\perp\pi})\;\delta(\hat{s}+\hat{t}+\hat{u})\\
&\times\;\left\{F_{+-}^{+-}(x_a,k_{\perp a})\,\cos[\phi_a+\phi_c''-\xi_a-\tilde{\xi}_a+\xi_c+\tilde{\xi}_c+\phi_\pi^H]\right.\\
&-\;\left.F_{-+}^{+-}(x_a,k_{\perp a})\,\cos[\phi_a-\phi_c''+\xi_a+\tilde{\xi}_a-\xi_c-\tilde{\xi}_c-\phi_\pi^H]\right\}\\
\times\;\hat{f}_{b/B}(x_b,k_{\perp b})\;&\left[\hat{M}_1^0\hat{M}_2^0(x_a,x_b,z;\boldsymbol{k}_{\perp a},\boldsymbol{k}_{\perp b},\boldsymbol{k}_{\perp\pi})\right]_{q_a b\to q_c d}\;\left[-2iD_{+-}^{\pi}(z,k_{\perp\pi})\right]\,.
\end{aligned}$$

A few comments are in order.

- All angles appearing in Eq. (56) can be expressed in terms of the pp c.m. integration variables, via Eqs. (33), (35)–(42) and (45).
- From Eqs. (48) and (49) one can see that D_{+-}^{π} is a purely imaginary quantity. The *Collins fragmentation function* [6, 15, 24, 27]

  $$-2iD_{+-}^{\pi} = 2\,\mathrm{Im}D_{+-}^{\pi}\equiv\Delta^N D_{\pi/q^\uparrow}\,, \qquad (57)$$

  has a simple interpretation in the frame in which the quark moves along the Z direction, with spin parallel ($q^\uparrow$) or antiparallel ($q^\downarrow$) to the Y-axis, while the $q\to\pi X$ process occurs in the XZ plane: it gives the difference between the number density of pions resulting from the fragmentation of a quark $q^\uparrow$

and a quark $q^\downarrow$. In the pp c.m. frame the quark transverse spin direction is not, in general, orthogonal to the $q \to \pi X$ plane and this reflects into the ϕ_π^H dependence appearing in Eq. (56).

- The product of elementary amplitudes $\hat{M}_1^0 \hat{M}_2^0$, see Eqs. (51) and (52), is, in a frame in which the partonic c.m. scattering plane is XZ, simply related to the spin transfer cross section:

$$\frac{1}{16\pi \hat{s}^2} \left[\hat{M}_1^0 \hat{M}_2^0\right]_{qb} = \frac{d\hat{\sigma}^{q^\uparrow b \to q^\uparrow b}}{d\hat{t}} - \frac{d\hat{\sigma}^{q^\uparrow b \to q^\downarrow b}}{d\hat{t}} \cdot \tag{58}$$

  Again, the parton intrinsic motions give, in general, more complicated, non planar configurations for the elementary scatterings, which induce dependences on the angles ξ_j, ξ'_j, η'_j and ϕ''_c.

- The distribution terms $F_{+-}^{+-}(x_a, k_{\perp a})$ and $F_{-+}^{+-}(x_a, k_{\perp a})$ are related to the distribution of transversely polarised quarks inside a transversely polarised proton; these transverse directions can be different for protons and quarks [25]. Without any intrinsic motion, only the $F_{+-}^{+-}(x_a)$ distribution would be present, coinciding with the transversity distribution $h_1(x_a)$ [24].

- Note that if one takes into account intrinsic motions only in the fragmentation process, assumed to occur in the XZ plane [$\boldsymbol{k}_{\perp a} = \boldsymbol{k}_{\perp b} = 0$, $(\boldsymbol{k}_{\perp \pi})_y = 0$, which implies all phases to be zero], one recovers the expression for the numerator of A_N (aside from the factor J) used in Refs. [15, 16].

We can now use Eqs. (56) and (4) to compute the SSA $A_N = (d\sigma^\uparrow - d\sigma^\downarrow)/2\, d\sigma$.

3. Attempts to fit the data: suppression of the Collins mechanism

As noted earlier, it was previously believed that the remarkably large SSA found *e.g.* in the E704 experiment [19] could be generated by either the Sivers [14] or the Collins mechanisms [15, 16]. However, to avoid handling the very complex kinematics and having to deal numerically with 8-dimensional integrals, only the one essential intrinsic $\boldsymbol{k}_\perp$, responsible for the asymmetry, was taken into account in these studies. We now believe that the phases involved, when the kinematics is treated carefully, are crucial, and, as we shall see, lead to a large suppression of the asymmetry due to the Collins mechanism. As explained in [4] there is little or no suppression of the asymmetry due to the Sivers mechanism.

In order to demonstrate the extent of the suppression we shall choose for the unmeasured soft functions in Eq. (56) their known upper bounds. Let us first write these functions with the notations of Refs. [11] and [24] (details will be given in [25]):

$$F_{+-}^{+-}(x, k_\perp) \quad = \quad h_1(x, k_\perp) = h_{1T}(x, k_\perp) + \frac{k_\perp^2}{2M_p^2}\, h_{1T}^\perp(x, k_\perp) \tag{59}$$

$$F^{+-}_{-+}(x, k_\perp) = \frac{k_\perp^2}{2M_p^2}\, h_{1T}^\perp(x, k_\perp) \tag{60}$$

$$-2iD^\pi_{+-}(z, k_\perp) = \Delta^N D_{\pi/q^\uparrow}(z, k_\perp) = \frac{2k_\perp}{zM_\pi}\, H_1^{\perp q}(z, k_\perp)\,. \tag{61}$$

where M_p and M_π are respectively the proton and pion mass. The following positivity bounds hold [28, 29]:

$$|h_1(x, k_\perp)| \leq \frac{1}{2}\,[q(x, k_\perp) + \Delta q(x, k_\perp)] = q_+(x, k_\perp) \tag{62}$$

$$\frac{k_\perp^2}{2M_p^2}\,|h_{1T}^\perp(x, k_\perp)| \leq \frac{1}{2}\,[q(x, k_\perp) - \Delta q(x, k_\perp)] = q_-(x, k_\perp) \tag{63}$$

$$|\Delta^N D_{\pi/q^\uparrow}(z, k_\perp)| \leq 2D_{\pi/q}(z, k_\perp)\,. \tag{64}$$

In our numerical estimates we adopt for all the unmeasured soft functions the above maximum possible values, and, moreover, adjust their signs so that the contributions from the valence flavours (up and down) reinforce each other in the π^+ reaction, producing a maximally large positive $A_N^{\pi^+}$. By isospin invariance it then turns out that this choice also produces a maximally large negative $A_N^{\pi^-}$. To be precise, we have computed the SSA, $A_N = (d\sigma^\uparrow - d\sigma^\downarrow)/2\,d\sigma$, via Eqs. (56) and (4), with the following choices:

- For the transversity pdf $F^{+-}_{+-}(x, k_\perp) = h_1(x, k_\perp)$ and its companion $h_{1T}^\perp$ we have only considered up and down quark flavours, without any sea contribution. We have saturated Eqs. (62) and (63):

$$h_1^u(x, k_\perp) = u_+(x, k_\perp) \qquad h_1^d(x, k_\perp) = -d_+(x, k_\perp) \tag{65}$$

$$\frac{k_\perp^2}{2M_p^2}\, h_{1T}^{\perp u}(x, k_\perp) = -u_-(x, k_\perp) \qquad \frac{k_\perp^2}{2M_p^2}\, h_{1T}^{\perp d}(x, k_\perp) = +d_+(x, k_\perp)\,. \tag{66}$$

  One naturally expects, for valence quarks, positive values for h_1^u and negative ones for h_1^d; the relative signs between h_1 and $h_{1T}^\perp$ are chosen in order to maximise the sum of their contributions in Eq. (56). The x and $k_\perp$ dependences in the unpolarised and polarised pdf are factorised assuming the same Gaussian form as in Eq. (6), with $\sqrt{\langle k_\perp^2\rangle} = 0.8$ GeV/c [4]. For the x-dependence of the unpolarised pdf we have adopted the MRST01 set [30] and for the polarised pdf either the LSS01 set [31] or the LSS-BBS set [32], as two examples of very different choices. We have used the same QCD evolution scale as in Ref. [4].

- We have chosen the z-dependence of the Collins function in such a way as to maximise the effects. Let us consider the production of π^+'s: since the dominant partonic contribution at large x_F is $ug \to ug$, for which the product of elementary amplitudes $\hat{M}_1^0 \hat{M}_2^0$ is negative, see Eqs. (55), in order to get a positive A_N we need a negative u-quark Collins function. That is, we satisfy the positivity bound (64) with:

$$\Delta^N D_{\pi^+/u^\uparrow}(z, k_\perp) = -2D_{\pi^+/u}(z, k_\perp)\,. \tag{67}$$

We consider here also the contribution of the sub-leading channel $dg \to dg$ (neglected in Refs. [15, 16]); as it enters with a negative h_1^d, in order to add all contributions, we use for the non-leading Collins function

$$\Delta^N D_{\pi^+/d^\uparrow}(z, k_\perp) = +2D_{\pi^+/d}(z, k_\perp)\,. \tag{68}$$

In this way also A_N for π^-'s is maximised in size (by isospin invariance).

For π^0's we take, exploiting isospin symmetry,

$$\Delta^N D_{\pi^0/q^\uparrow} = \frac{1}{2}\left(\Delta^N D_{\pi^+/u^\uparrow} + \Delta^N D_{\pi^+/d^\uparrow}\right) = \frac{1}{2}\left(-2D_{\pi^+/u} + 2D_{\pi^+/d}\right)\,, \tag{69}$$

where $q = u, \bar{u}, d, \bar{d}$ and which still fulfills the bound (64). The z and $k_\perp$ dependences of the unpolarised fragmentation functions are also factorised, with the same Gaussian dependence as in Ref. [4], which introduces a z-dependent $\langle k_\perp^2\rangle$ value, smaller than the constant $\langle k_\perp^2\rangle$ value assumed for the pdf. This value allows a good understanding of the unpolarised cross sections; we have explicitly checked that increasing the ff $\langle k_\perp^2\rangle$ does not change significantly our present results (while spoiling the agreement with the unpolarised cross sections). The z-dependent unpolarised ff are taken either from Kretzer [33] or from KKP [34, 4], as typical examples of two different sets.

With the above choices, Eqs. (59)–(69), we can (over)estimate the maximum value that, within our approach, the Collins mechanism alone contributes to the SSA in $p^\uparrow p \to \pi X$ processes. The results are presented in the four plots of Fig. 1, which show $(A_N)^{\rm Collins}_{\rm max}$ as a function of x_F, at $p_T = 1.5$ GeV/c and $\sqrt{s} \simeq 19.4$ GeV: this is the E704 kinematical region and a comparison with their data [19] is shown. The only difference between the plots is given by different choices of the polarised distribution functions and/or the unpolarised fragmentation functions. Four different combinations are possible: two different sets of polarised pdf, LSS01 [31] or LSS-BBS [32], and two different sets of unpolarised ff, Kretzer [33], or KKP [34]. The four combinations exhaust all possible features of choices available in the literature. The results clearly show that the Collins mechanism alone, even maximising all its effects, cannot explain the observed SSA values; its contribution, when all proper phases are taken into account, fails to explain the large E704 values observed for $A_N^{\pi^+}$ and $A_N^{\pi^-}$ at large x_F.

4. Comments and conclusions

We have developed a consistent formalism to describe, within pQCD and a factorisation scheme, the inclusive production of particles in hadronic high energy collisions; all intrinsic motions of partons in hadrons and of hadrons in fragmenting partons, are properly taken into account. Such a scheme has been applied, in a parallel paper [4], to the description of several unpolarised cross sections and to the computation of SSA in $p^\uparrow p \to \pi X$ processes, generated by the Sivers mechanism alone. In this paper we have again considered SSA in $p^\uparrow p \to \pi X$ processes, but focussing on the contribution of the Collins mechanism alone. Previous work [14, 15, 16], performed in a similar scheme with simplified kinematics, showed that both the Collins and the Sivers mechanisms, could alone explain the observed data on SSA.

Such a conclusion has now to be modified: while properly chosen Sivers distribution functions could still explain the data [4], there are no Collins fragmentation functions able to do that, as Fig. 1 shows. The failure of the Collins mechanism, when all partonic motions are included, can be understood from the complicated azimuthal angle dependencies in Eq. (56): the many phases arising in polarised distribution and fragmentation functions, and in polarised non planar elementary dynamics conspire, when integrated, to strongly suppress the final result.

The situation with the Sivers contribution alone is much simpler, as the partons participating in the elementary dynamics and in the fragmentation process are not polarised. As a consequence, the phase structure of the numerator of A_N, in Sivers case, contains only one phase, the Sivers angle (see Eqs. (44) and (45) of Ref. [4]). Its integration, coupled with the dependence of the elementary dynamics on the same angle, does not significantly suppress the result. In this case, the simplified kinematics of Ref. [14] contains the main physical features of the mechanism and gives a reasonably accurate computation of A_N.

Our results show, once more, the importance and subtleties of spin effects; all phases have to be properly considered and they often play crucial and unexpected roles. The analysis of this paper will be extended to other processes, like semi-inclusive Deep Inelastic Scattering, where many SSA effects have been observed [17, 18] and are being measured.

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References

[1] PHENIX Collaboration, S.S. Adler *et al.*, *Phys. Rev. Lett.* **91** (2003) 241803; STAR Collaboration, J. Adams *et al.*, *Phys. Rev. Lett.* **92** (2004) 171801

[2] R.D. Field and R.P. Feynman, *Phys. Rev.* **D15** (1977) 2590; R.P. Feynman, R.D. Field and G.C. Fox, *Phys. Rev.* **D18** (1978) 3320

[3] X-N. Wang, *Phys. Rev.* **C61** (2000) 064910; C.-Y. Wong and H. Wang, *Phys. Rev.* **C58** (1998) 376; Y. Zhang, G. Fai, G. Papp, G.G. Barnaföldi and P. Lévai, *Phys. Rev.* **C65** (2002) 034903

[4] U. D'Alesio and F. Murgia, e-Print Archive: hep-ph/0408092

[5] R.N. Cahn, *Phys. Lett.* **B78** (1978) 269, *Phys. Rev.* **D40** (1989) 3107

[6] J.C. Collins, *Nucl. Phys.* **B396** (1993) 161

[7] J.C. Collins, D.E. Soper and G. Sterman, *Nucl. Phys.* **B250** (1985) 199; J.C. Collins and D.E. Soper, *Nucl. Phys.* **B193** (1981) 381

[8] X. Ji, J.-P. Ma and F. Yuan, e-Print Archive: hep-ph/0404183; *Phys. Lett.* **B597** (2004) 299

[9] D. Sivers, *Phys. Rev.* **D41** (1990) 83; **D43** (1991) 261

[10] D. Boer, *Phys. Rev.* **D60** (1999) 014012

[11] P.J. Mulders and R.D. Tangerman, *Nucl. Phys.* **B461** (1996) 197; *Nucl. Phys.* **B484** (1997) 538 (E)

[12] M. Anselmino, D. Boer, U. D'Alesio and F. Murgia, *Phys. Rev.* **D63** (2001) 054029

[13] E. Leader, e-Print Archive: hep-ph/0405284

[14] M. Anselmino, M. Boglione and F. Murgia, *Phys. Lett.* **B362** (1995) 164; M. Anselmino and F. Murgia, *Phys. Lett.* **B442** (1998) 470

[15] M. Anselmino, M. Boglione and F. Murgia, *Phys. Rev.* **D60** (1999) 054027

[16] M. Boglione and E. Leader, *Phys. Rev.* **D61** (2000) 114001

[17] HERMES Collaboration, A. Airapetian *et al.*, *Phys. Rev. Lett.* **84** (2000) 4047; *Phys. Rev.* **D64** (2001) 097101

[18] Talk by N. Makins on behalf of HERMES Collaboration, Transversity Workshop, Athens, Greece, October 6-7, 2003

[19] D.L. Adams *et al.*, *Phys. Lett.* **B264** (1991) 462; A. Bravar *et al.*, *Phys. Rev. Lett.* **77** (1996) 2626

[20] J. Adams *et al.*, *Phys. Rev. Lett.* **92** (2004) 171801

[21] J.C. Collins, *Nucl. Phys.* **B394** (1993) 169; J.C. Collins, S.F. Heppelmann and G.A. Ladinsky, *Nucl. Phys.* **B420** (1994) 565

[22] For a pedagogical introduction to all the basics of helicity formalism, see, *e.g.*, E. Leader, *Spin in Particle Physics*, Cambridge University Press, 2001

[23] D. Boer, P. Mulders and F. Pijlman, *Nucl. Phys.* **B667** (2003) 201

[24] V. Barone, A. Drago and P. Ratcliffe, *Phys. Rep.* **359** (2002) 1

[25] M. Anselmino, M. Boglione, U. D'Alesio, E. Leader, S. Melis and F. Murgia, in preparation

[26] M. Anselmino, U. D'Alesio and F. Murgia, *Phys. Rev.* **D67** (2003) 074010

[27] D. Boer and P. Mulders, *Phys. Rev.* **D57** (1998) 5780

[28] J. Soffer, *Phys. Rev. Lett.* **74** (1995) 1292

[29] A. Bacchetta, M. Boglione, A. Henneman and P.J. Mulders, *Phys. Rev. Lett.* **85** (2000) 712

[30] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Phys. Lett.* **B531** (2002) 216

[31] E. Leader, A.V. Sidorov and D.B. Stamenov, *Eur. Phys. J.* **C23** (2002) 479

[32] E. Leader, A.V. Sidorov and D.B. Stamenov, *Int. J. Mod. Phys.* **A13** (1998) 5573

[33] S. Kretzer, *Phys. Rev.* **D62** (2000) 054001

[34] B.A. Kniehl, G. Kramer, and B. Pötter, *Nucl. Phys.* **B582** (2000) 514

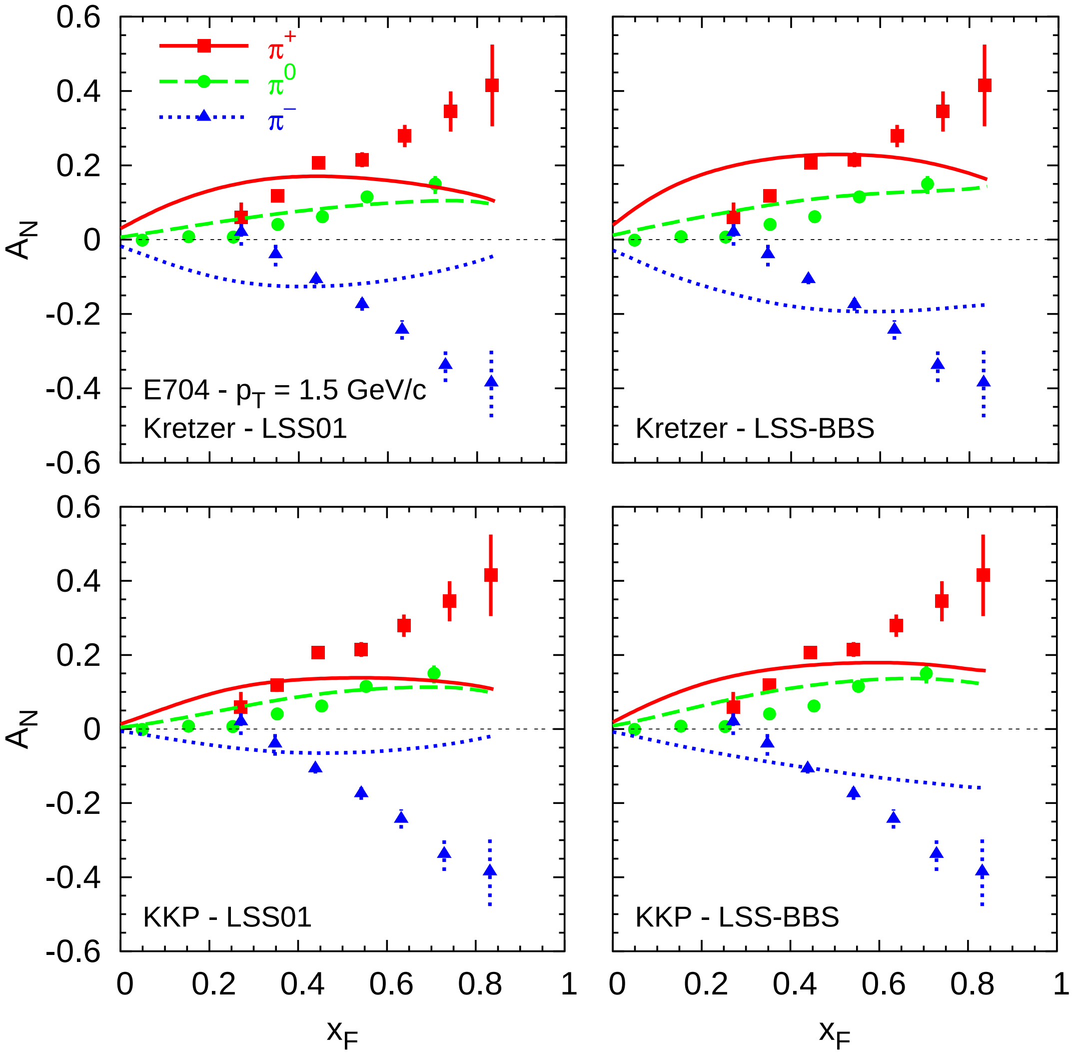

Figure 1: Maximised values of A_N vs. x_F, at $\sqrt{s} \simeq 19.4$ GeV and fixed $p_T = 1.5$ GeV$/c$, as given by the Collins mechanism alone; the values shown are obtained by saturating all bounds on the unknown soft functions and by adding constructively all different contributions. The four plots correspond to different choices of the distribution and fragmentation functions used to saturate the bounds, as indicated in the legends. In each plot the upper, middle and lower sets of curves and data refer respectively to π^+, π^0 and π^-. Data are from Ref. [19]. See the text for further details.