The longitudinal spin structure of the nucleon

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Summary. — We review first the parton model formalism for polarized deep inelastic lepton-hadron scattering. Topics discussed include the "spin crisis in the parton model", the role of the axial anomaly, our knowledge of the polarized gluon number density and attempts to measure it. Secondly, going beyond the simple parton model, we discuss the evolution of parton densities, the generalization of the parton model in QCD, perturbative QCD corrections and scheme dependence. Finally we comment on our knowledge of the polarized strange quark density and attempts to learn about it from semi-inclusive deep inelastic scattering.

1. – Deep inelastic scattering

Deep inelastic lepton-hadron scattering (DIS) has played a seminal role in the development of our present understanding of the sub-structure of elementary particles. The discovery of Bjorken scaling in the late nineteen-sixties provided the critical impetus for the idea that elementary particles contain almost pointlike constituents and for the subsequent invention of the Parton Model. DIS continued to play an essential role in the long period of consolidation that followed, in the gradual linking of partons and quarks, in the discovery of the existence of missing constituents, later identified as gluons, and in

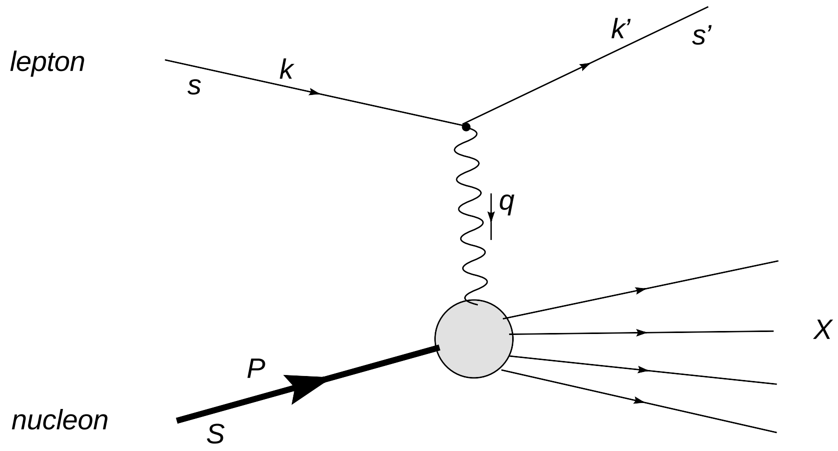

Fig. 1. – Feynman diagram for deep inelastic lepton-hadron scattering

the wonderful confluence of all the different parts of the picture into a coherent dynamical theory of quarks and gluons – Quantum Chromodynamics (QCD).

1·1. *General formalism in one photon exchange approximation.* – Consider the inelastic scattering of polarized leptons on polarized nucleons. We denote by m the lepton mass, k (k') the initial (final) lepton four-momentum and s (s') its covariant spin four-vector, such that $s \cdot k = 0$ ($s' \cdot k' = 0$) and $s \cdot s = -1$ ($s' \cdot s' = -1$); the nucleon mass is M and the nucleon four-momentum and spin four-vector are, respectively, P and S. Assuming one photon exchange, (see fig. 1), the differential cross-section for detecting the final polarized lepton in the solid angle $d\Omega$ and in the final energy range $(E',\ E'+dE')$ in the laboratory frame, $P = (M, \mathbf{0}),\ k = (E, \boldsymbol{k}),\ k' = (E', \boldsymbol{k}')$, can be written as

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu} \tag{1}$$

where $q = k - k'$ and α is the fine structure constant.

The leptonic tensor $L_{\mu\nu}$ is given by

$$\begin{aligned} &L_{\mu\nu}(k,s;k',) = \\ &\sum_{s'} [\bar{u}(k',s')\ \gamma_\mu\ u(k,s)]^*\ [\bar{u}(k',s')\ \gamma_\nu\ u(k,s)] \end{aligned} \tag{2}$$

and can be split into symmetric (S) and antisymmetric (A) parts under μ, ν interchange:

$$L_{\mu\nu}(k,s;k',) = 2\{L^{(S)}_{\mu\nu}\ (k;k') + iL^{(A)}_{\mu\nu}\ (k,s;k')\} \tag{3}$$

where

$$
\begin{aligned}
L^{(S)}_{\mu\nu}\ (k;k') &= k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}\ (k\cdot k' - m^2) \\
L^{(A)}_{\mu\nu}\ (k,s;k') &= m\ \varepsilon_{\mu\nu\alpha\beta}\ s^\alpha\ q^\beta
\end{aligned} \tag{4}
$$

The unknown hadronic tensor $W_{\mu\nu}$ describes the interaction between the virtual photon and the nucleon and depends upon four scalar structure functions, the unpolarized functions $W_{1,2}$ and the spin-dependent functions $G_{1,2}$. These must be measured and can then be studied in theoretical models, in our case in the QCD-modified parton model. These can only be functions of the scalars q^2 and $q \cdot P$. Usually people work with

$$
Q^2 \equiv -q^2 \quad \text{and} \quad x_{Bj} \equiv Q^2/2q\cdot P = Q^2/2M\nu \tag{5}
$$

where $\nu = E - E'$ is the energy of the virtual photon in the Lab frame.
x_{Bj} is known as "x-Bjorken", and we shall simply write it as x.
One has:

$$
W_{\mu\nu}(q;P,S) = W^{(S)}_{\mu\nu}(q;P) + i\ W^{(A)}_{\mu\nu}(q;P,S) \tag{6}
$$

with

$$
\begin{aligned}
&\frac{1}{2M}\ W^{(S)}_{\mu\nu}(q;P) = \\
&\quad \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\ W_1(P\cdot q, q^2) \\
&+ \left[\left(P_\mu - \frac{P\cdot q}{q^2}\ q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\ q_\nu\right)\right]\frac{W_2(P\cdot q,q^2)}{M^2}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
&\frac{1}{2M}\ W^{(A)}_{\mu\nu}(q;P,S) = \\
&\quad \varepsilon_{\mu\nu\alpha\beta}\ q^\alpha \left\{ M S^\beta G_1(P\cdot q, q^2) \right. \\
&+ \left. [(P\cdot q)S^\beta - (S\cdot q)P^\beta]\ \frac{G_2(P\cdot q, q^2)}{M} \right\}.
\end{aligned} \tag{8}
$$

Note that these expressions are electromagnetic gauge-invariant:

$$
q^\mu W_{\mu\nu} = 0 \tag{9}
$$

From these one has

$$\frac{d^2\sigma}{d\Omega\ dE'} = \frac{\alpha^2}{2Mq^4}\ \frac{E'}{E}\ \left[L^{(S)}_{\mu\nu}\ W^{\mu\nu(S)} - L^{(A)}_{\mu\nu}\ W^{\mu\nu(A)}\right] \tag{10}$$

Differences of cross-sections with opposite target spins single out the $L^{(A)}_{\mu\nu}\ W^{\mu\nu(A)}$ term:

$$\begin{aligned} &\left[\frac{d^2\sigma}{d\Omega\ dE'}(k,s,P,-S;k') - \frac{d^2\sigma}{d\Omega\ dE'}\ (k,s,P,S;k')\right] \\ &= \frac{\alpha^2}{2Mq^4}\ \frac{E'}{E}\ 4L^{(A)}_{\mu\nu}\ W^{\mu\nu(A)} \end{aligned} \tag{11}$$

After some algebra (for a detailed explanation of the steps involved, see [1]) one obtains:

$$\begin{aligned} &\frac{d^2\sigma^{s,S}}{d\Omega\ dE'} - \frac{d^2\sigma^{s,-S}}{d\Omega\ dE'} \quad = \\ &\frac{8m\alpha^2 E'}{q^4 E}\left\{\left[(q\cdot S)(q\cdot s) + Q^2(s\cdot S)\right] MG_1\right. \\ &\left. +Q^2\Big[(s\cdot S)(P\cdot q) - (q\cdot S)(P\cdot s)\Big]\frac{G_2}{M}\right\} \end{aligned} \tag{12}$$

which yields information on the polarized structure functions $G_1(P\cdot q, q^2)$ and $G_2(P\cdot q, q^2)$.

In the Bjorken limit, or Deep Inelastic Scattering (DIS) regime,

$$\begin{aligned} -q^2 = Q^2 \to \infty \qquad & \nu = E - E' \to \infty \\ & x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2M\nu}\ , \text{ fixed} \end{aligned}$$

the scalar functions are known to approximately scale:

$$\begin{aligned} \lim_{Bj}\ MW_1(P\cdot q, Q^2) &= F_1(x) \\ \lim_{Bj}\ \nu W_2(P\cdot q, Q^2) &= F_2(x)\,, \end{aligned} \tag{13}$$

$$\begin{aligned} \lim_{Bj}\ \frac{(P\cdot q)^2}{\nu}\ G_1(P\cdot q, Q^2) &= g_1(x) \\ \lim_{Bj}\ \nu\ (P\cdot q)\ G_2(P\cdot q, q^2) &= g_2(x)\,. \end{aligned} \tag{14}$$

where $F_{1,2}$ and $g_{1,2}$ vary very slowly with Q^2 at fixed x.

In terms of $g_{1,2}$ the expression for $W_{\mu\nu}^{(A)}$ becomes

$$W_{\mu\nu}^{(A)}(q;P,s) = \frac{2M}{P\cdot q}\,\varepsilon_{\mu\nu\alpha\beta}\,q^\alpha\Bigg\{S^\beta g_1(x,Q^2) + \left[S^\beta - \frac{(S\cdot q)\,P^\beta}{(P\cdot q)}\right] g_2(x,Q^2)\Bigg\}. \tag{15}$$

1˙2. *Polarized DIS*. – The cross-section for unpolarized scattering is given by

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}[xy^2F_1 + (1-y)F_2] \tag{16}$$

where we have used

$$y \equiv \frac{\nu}{E} = \frac{P\cdot q}{P\cdot k} \tag{17}$$

and where $s = (P+k)^2$. If now we take the lepton and target nucleon polarized longitudinally, i.e. along or opposite to the direction of the lepton beam, then, under reversal of the nucleon's spin direction the cross-section difference is given by

$$\frac{d^2\sigma^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy} - \frac{d^2\sigma^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} = \frac{16\pi\alpha^2}{Q^2}\left[(1-\frac{y}{2})\,g_1 - \frac{2M^2xy}{Q^2}g_2\right]. \tag{18}$$

For nucleons polarized transversely in the scattering plane, one finds

$$\frac{d^2\sigma^{\rightarrow\Uparrow}}{dx\,dy} - \frac{d^2\sigma^{\rightarrow\Downarrow}}{dx\,dy} = -\frac{16\alpha^2}{Q^2}\,\left(\frac{2Mx}{Q}\right)\sqrt{1-y}\,\left[\frac{y}{2}g_1 + g_2\right]. \tag{19}$$

In principle these allow measurement of both g_1 and g_2, but the transverse asymmetry is much smaller and therefore much more difficult to measure. Only in the past few years has it been possible to gather information on g_2 which turns out to be smaller than g_1.

2. – The simple parton model

In a reference frame where the proton is moving very fast, say along the OZ axis, it can be viewed as a beam of *parallel-moving* partons, as shown in fig. 2.

In the hard interaction with the photon, the quark-partons are treated as free, massless particles with momentum $x'\boldsymbol{P}$, as shown in fig. 3.

One finds that the antisymmetric part of the hadronic tensor is given by

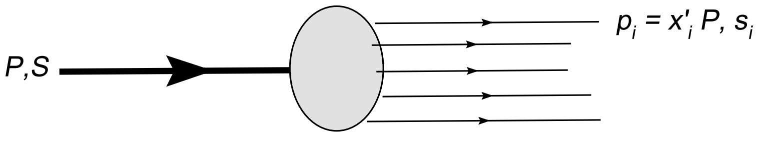

Fig. 2. – Visualization of parton density $q(x', s))$

$$W^{(A)}_{\mu\nu}(q:P,S) = \sum_{f,s} e_f^2\,\frac{1}{2P\cdot q}\int_0^1 \frac{dx'}{x'}\,\delta(x'-x)$$
$$n_f(x';s,S)\,w^{(A)}_{\mu\nu}(x';q,s) \tag{20}$$

where $w^{(A)}_{\mu\nu}(x';q,s)$ is the quark tensor and is just like the leptonic tensor $L^{(A)}_{\mu\nu}$ since the quarks are treated as point-like particles, and the sum is over flavours f and spin orientations s of the struck quark.

The delta-function that forces $x' = x$ arises from the usual convention of treating the quarks as "free" particles on mass shell i.e. one takes

$$p^2 = (x'P)^2 = 0 \qquad (q+p)^2 = (q+x'P)^2 = 0 \tag{21}$$

so that

$$q^2 + 2x'q\,\boldsymbol{.}\,P = 0 \tag{22}$$

With the definition of $x_{Bjorken}$ [see eq. (5)] this implies

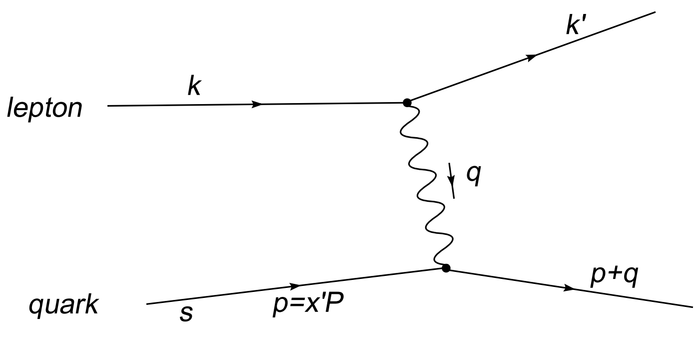

Fig. 3. – Lepton-quark deep inelastic scattering

$$-Q^2 + Q^2 \frac{x}{x'} = 0 \qquad \text{or} \qquad x' = x \tag{23}$$

However, let us for the moment take $p^2 = m^2$ and $p'^2 = m'^2$. One finds that

$$w_{\mu\nu}^{(A)} = 2\, \varepsilon_{\mu\nu\alpha\beta}\, m' s^\alpha \left[(1 - \frac{m}{m'})\, p^\beta - \frac{m}{m'}\, q^\beta \right] \tag{24}$$

Note that because of the term in round brackets the result is not gauge invariant i.e. $q^\mu\, w_{\mu\nu} \neq 0$ *unless* $m' = m$. But for *longitudinal* polarization, $s^\alpha = s_L^\alpha$, we have

$$m'\, s_L^\alpha \rightarrow \pm p^\alpha \qquad \text{for} \qquad \frac{m'}{p} \ll 1 \tag{25}$$

and therefore the non-gauge invariant term vanishes because of the antisymmetry of the ε symbol.

2˙1. *Longitudinal polarization.* – Consider a fast moving proton, momentum along OZ, and polarized along OZ. Substituting eq. (24) into eq. (20), and comparing with eq. (15), we find

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \,\triangle q_f(x) \tag{26}$$

where

$$\triangle q(x) = q_{(+)}(x) - q_{(-)}(x) \tag{27}$$

where $q_{(\pm)}(x)$ are the number densities of quarks whose spin orientation is parallel or antiparallel to the spin direction of the proton (see fig. 4). In terms of these, the usual (unpolarized) parton density is

$$q(x) = q_{(+)}(x) + q_{(-)}(x) \tag{28}$$

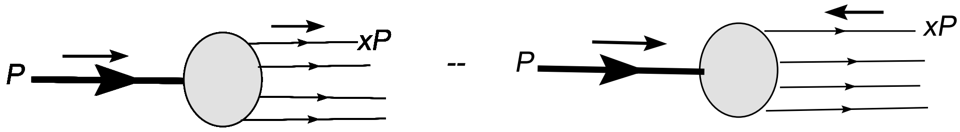

Fig. 4. – Visualization of the longitudinally polarized parton density. The upper arrows show the spin direction $\Delta q(x)$

2˙2. *What about* $g_2(x)$*?*. – For a transversely polarized quark we have seen eq. (24), that the quark tensor is only gauge-invariant if $m' = m$. This is a bad sign! The result should not be sensitive to the precise value of the quark mass. So what happens if we take $m' = m$? We find $g_2(x) = 0$! Is this result reliable? Not in so far as it relates to the proton. The point is that the quark DIS tensor does not have a transverse asymmetry, so we cannot hope to use it to provide such an asymmetry in the proton.

There are many different, inconsistent results for $g_2(x)$ in the literature, including this beautiful one

$$g_2(x) = \frac{1}{2} \sum e_f^2 \left(\frac{m_q}{xM} - 1\right) \Delta q(x) \tag{29}$$

due to Anselmino and myself [2] , which, alas, should not be taken seriously.

The only reliable result is the Wandzura-Wilcczek relation [3]

$$g_2(X) \simeq -g_1(x) + \int_x^1 \frac{g_1(x')}{x'}\, dx' \tag{30}$$

which was originally derived as an approximation in an operator product expansion approach, but which has recently been shown to be derivable directly in the simple parton model [4] .

3. – The spin crisis in the parton model

The accepted expression for g_1 was completely analogous to the equation for F_1, with the unpolarized quark density replaced by the (longitudinal) polarized density $\Delta q(x)$.

The expression for g_1 is then:

$$g_1(x) = \frac{1}{2}\left\{\frac{4}{9}\Delta u(x) + \frac{1}{9}\Delta d(x) + \frac{1}{9}\Delta s(x) + \text{antiquarks}\right\} \tag{31}$$

Define combinations of quark densities which have specific transformation properties under the group of flavour transformations $SU(3)_F$:

$$\Delta q_3 = (\Delta u + \Delta\overline{u}) - (\Delta d + \Delta\overline{d}) \tag{32}$$

$$\Delta q_8 = (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) - 2(\Delta s + \Delta\overline{s}) \tag{33}$$

$$\Delta\Sigma = (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) + (\Delta s + \Delta\overline{s}) \tag{34}$$

which transform respectively as the third component of an isotopic spin triplet, the eighth component of an $SU(3)_F$ octet and a flavour singlet. Then

$$g_1(x) = \frac{1}{9}\left[\frac{3}{4}\Delta q_3(x) + \frac{1}{4}\Delta q_8(x) + \Delta\Sigma\right] \tag{35}$$

Taking the first moment of this yields

$$\Gamma_1 \equiv \int_0^1 g_1(x)dx = \frac{1}{12}\left[a_3 + \frac{1}{\sqrt{3}}a_8 + \frac{4}{3}a_0\right] \tag{36}$$

where

$$\begin{aligned} a_3 &= \int_0^1 dx\ \Delta q_3(x) \\ a_8 &= \frac{1}{\sqrt{3}}\int_0^1 dx\ \Delta q_8(x) \\ a_0 &= \Delta\Sigma \equiv \int_0^1 dx\ \Delta\Sigma(x) \end{aligned} \tag{37}$$

Via the Operator Product Expansion these moments can be related to hadronic matrix elements of currents which are *measurable* in *other* processes, as will be explained below.

The hadronic tensor $W^{\mu\nu}$ is given by the Fourier transform of the nucleon matrix elements of the commutator of electromagnetic currents $J_\mu(x)$:

$$W_{\mu\nu}(q; P, S) = \frac{1}{2\pi}\int d^4x\ e^{iq\cdot x}\langle P, S|[J_\mu(x), J_\nu(0)]|P, S\rangle \tag{38}$$

where S^μ is the covariant spin vector specifying the nucleon state of momentum P^μ.

In hard processes, $x^2 \simeq 0$ is important, so we can use the Wilson expansion.

The OPE gives moments of $g_{1,2}$ in terms of hadronic matrix elements of certain operators multiplied by perturbatively calculable coefficient functions. The a_i in Eq. (36) are hadronic matrix elements of the octet of quark $SU(3)_F$ axial-vector currents $J^j_{5\mu}$ ($j = 1, ..., 8$) and the flavour singlet axial current $J^0_{5\mu}$.

The octet currents are

$$J^j_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\left(\frac{\lambda_j}{2}\right)\psi \qquad (j = 1, 2, ..., 8) \tag{39}$$

where the λ_j are the usual Gell-Mann matrices and ψ is a column vector in flavour space

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix}, \tag{40}$$

and the flavour singlet current is

$$J^0_{5\mu} = \bar{\boldsymbol{\psi}}\gamma_\mu\gamma_5\boldsymbol{\psi}\,. \tag{41}$$

The forward matrix elements of the $J^j_{5\mu}$ can only be proportional to S^μ, and the a_j are defined by

$$\begin{aligned} \langle P,S|J^j_{5\mu}|P,S\rangle &= Ma_jS_\mu \\ \langle P,S|J^0_{5\mu}|P,S\rangle &= 2Ma_0S_\mu\,. \end{aligned} \tag{42}$$

Analogous to Eq. (39) one introduces an octet of vector currents

$$J^j_\mu = \bar{\psi}\gamma_\mu\left(\frac{\lambda_\mathbf{j}}{2}\right)\psi \qquad (\mathbf{j} = \mathbf{1},...,\mathbf{8}) \tag{43}$$

which are *conserved currents* to the extent that $SU(3)_F$ is a symmetry of the strong interactions.

These octets of currents control the β-decays of the neutron and of the octet of hyperons which implies that the values of a_3 and a_8 are known from *other* measurements.

Therefore a measurement of Γ_1 can be considered as giving the value of the flavour singlet a_0. Now the European Muon Collaboration, working at CERN, measured the first moment of the spin dependent structure function g_1 of the proton and in 1988 announced their startling results [5].

Knowing the values of a_3 and a_8, the EMC measurement implied

$$a_0^{EMC} \simeq 0 \tag{44}$$

But in the naive parton model

$$a_0 = \Delta\Sigma \tag{45}$$

where $\Delta\Sigma$ is given by eq. (34).

In 1974 Ellis and Jaffe [6] had suggested that one could ignore the contribution from the strange quark i.e. from $\Delta s + \Delta\bar{s}$, implying that

$$a_0 \simeq a_8 \simeq 0.59 \tag{46}$$

Thus the EMC result eq. (44) is in gross contradiction with Ellis-Jaffe.

It was this contradiction which at first aroused interest in the EMC result, but it was soon realized that their result had far more serious consequences.

Consider the physical significance of $\Delta\Sigma(x)$. Since $q_\pm(x)$ count the number of quarks of momentum fraction x with spin component $\pm\frac{1}{2}$ along the direction of motion of the proton (say the z-direction), the total contribution to J_z coming from a given flavour quark is

$$S_z = \int_0^1 dx \left\{ \left(\frac{1}{2}\right) q_+(x) + \left(\frac{-1}{2}\right) q_-(x) \right\}$$
$$= \frac{1}{2} \int_0^1 dx\ \Delta q(x)\,. \tag{47}$$

It follows that

$$a_0 = 2S_z^{quarks} \tag{48}$$

where S_z^{quarks} is the contribution to J_z from the spin of all quarks and antiquarks.

Note that a_0 *plays two roles*:

i) it measures the z component of the spin carried by the quarks

ii) it measures the expectation value of the flavour singlet axial-vector current.

What is the connection? Noether's Theorem tells us that the spin density operator for a spin 1/2 particle is $1/2\bar{\psi}(x)\gamma^\rho\sigma^{\nu\lambda}\psi(x)$.

Having the spin in the z direction implies $\rho = 0, \nu = 1, \lambda = 2$. Then we recognize the connection between the operators

$$1/2\bar{\psi}(x)\gamma^0\sigma^{12}\psi(x) = 1/2\bar{\boldsymbol{\psi}}\gamma_3\gamma_5\boldsymbol{\psi} \tag{49}$$

If we write the nucleon state as a superposition of partonic states, we find that

$$\text{(axial-vector current)}_{\text{expectation value}}$$
$$= 2 \times \text{(spin carried by quarks)}$$

3˙1. *Simple parton model.* – One takes $\mathbf{p}_\perp = \mathbf{0}$ and all quarks move parallel to the parent hadron.

Thus the quark, momentum $\mathbf{p}$, has $\mathbf{p} = \mathbf{xP}$ which implies that the orbital angular momentum carried by quarks is perpendicular to $\mathbf{P}$, and hence does not contribute to J_z. Thus, in the simple parton model, one expects for a proton of helicity $+1/2$:

$$S_z^{quarks} = J_z = 1/2\,. \tag{50}$$

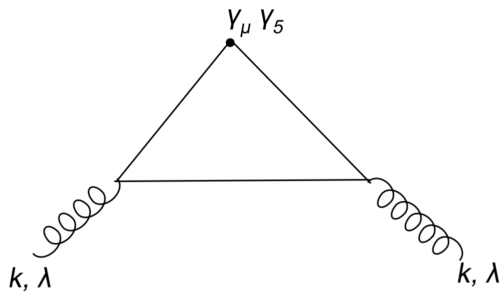

Fig. 5. – Feynman diagram responsible for the anomaly

The EMC result [5] for the value of a_0, on the contrary, implied that

$$\left(S_z^{quarks}\right)_{Exp} = 0.03 \pm 0.06 \pm 0.09\ . \tag{51}$$

It was this highly unexpected result which was termed a "spin crisis in the parton model" [7].

3·2. *Resolution of the spin crisis.* – Take the divergence of the flavour-f axial current. Using the equations of motion one finds

$$\partial^\mu J^f_{5\mu} = 2im_q \bar{\psi}_f(x)\,\gamma_5\,\psi_f(x) \tag{52}$$

where m_q is mass of the quark of flavour f.

In the chiral limit $m_q \to 0$ this implies that $J^f_{5\mu}$ is conserved. This would mean a symmetry between left and right-handed quarks and ultimately a parity degeneracy of the hadron spectrum *e.g.* there would exist two protons, of opposite parity.

Adler [8], and Bell and Jackiw [9] showed that the formal argument from the free equations of motion is not reliable, and there exists an anomalous contribution arising from the triangle diagram shown in fig. 5.

For the QCD case one finds

$$\partial^\mu J^f_{5\mu} = \frac{\alpha_s}{4\pi}\, G^a_{\mu\nu} \widetilde{G}^{\mu\nu}_a = \frac{\alpha_s}{2\pi}\, \mathrm{Tr}\,[\mathbf{G}_{\mu\nu}\ \widetilde{\mathbf{G}}^{\mu\nu}] \tag{53}$$

where α_s is QCD analogue of fine structure constant, and $\widetilde{G}^a_{\mu\nu}$ is the dual field tensor

$$\widetilde{G}^a_{\mu\nu} \equiv (1/2)\,\varepsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}_a \tag{54}$$

A field vector or tensor without a colour label stands for a matrix. In this case

$$\mathbf{G}_{\mu\nu} \equiv (\mathbf{1/2})\lambda_{\mathbf{a}} \mathbf{G}^{\mathbf{a}}_{\mu\nu}\,. \tag{55}$$

The above result is actually very tricky. It is a particular limit of a non-uniform function [1]. If we take $m_q \neq 0$, $k^2 \neq 0$ then the RHS of Eq. (53) is multiplied by

$$T(m_q^2/k^2) = 1 - \frac{2m_q^2/k^2}{\sqrt{1+4m_q^2/k^2}} \ln\left(\frac{\sqrt{1+4m_q^2/k^2}+1}{\sqrt{1+4m_q^2/k^2}-1}\right) \tag{56}$$

The anomaly corresponds to $T \to 1$ for $(m_q^2/k^2) \to 0$. But for on-shell gluons, $k^2 = 0$, and $m_q \neq 0$, *i.e.* in the limit $(m_q^2/k^2) \to \infty$ the terms cancel, $T \to 0$, and there is no anomaly. For gluons bound inside a nucleon one should utilize $k^2 \neq 0$ and the anomalous triangle contributes.

3˙3. *Effect of anomaly.* – Adler's expression for the triangle diagram, modified to QCD, gives for the forward *gluonic* matrix element of the flavour f current

$$\langle k,\lambda|J^f_{5\mu}|k,\lambda\rangle = -\frac{\alpha_s}{2\pi}\, S^g_\mu(k,\lambda)\, T(m_q^2/k^2) \tag{57}$$

where λ is the gluon helicity and

$$S^g_\mu(k,\lambda) \approx \lambda k_\mu \tag{58}$$

is the covariant spin vector for almost massless gluons.

We can now compute the gluonic contribution to the hadronic expectation value $\langle P,S|J^0_{5\mu}|P,S\rangle$. The gluons being bound will be slightly off-shell *i.e.* $k^2 \neq 0$, but small. The full triangle contribution involves a sum over all quark flavours.

Take m_u, m_d and m_s to be $\ll k^2$ whereas m_c, m_b and m_t are $\gg k^2$. The function $T(m_q^2/k^2)$ thus takes the values:

$$\begin{aligned} T &= 1 \quad \text{for} \quad u,\ d,\ s \\ T &= 0 \quad \text{for} \quad c,\ b,\ t \end{aligned} \tag{59}$$

Hence the gluon contribution is [10, 11, 12, 13]

$$\begin{aligned} a_0^{gluons}(Q^2) &= -3\,\frac{\alpha_s}{2\pi}\int_0^1 dx\ \Delta G(x,Q^2) \\ &\equiv -3\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2) \end{aligned} \tag{60}$$

where $\Delta G(x)$ is analogous to $\Delta q(x)$

$$\Delta G(x) = G_+(x) - G_-(x) \tag{61}$$

So, there exists a *gluonic* contribution to first moment of g_1 !

$$\Gamma_{1p}^{gluons}(Q^2) = -\frac{1}{3}\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2)\,. \tag{62}$$

This result is of fundamental importance.

It implies that the simple parton model formulae for a_0 (and hence for Γ_1^p) in terms of the Δq_f are incorrect

Instead,

$$a_0 = \Delta\Sigma - 3\,\frac{\alpha_s}{2\pi}\,\Delta G \tag{63}$$

The fundamental conclusion is *that the small measured value of* a_0 *does not necessarily imply that* $\Delta\Sigma$ *is small.*

3˙4. *A surprising aspect of this result!*. – The simple parton model is usually thought of as the limit when the QCD coupling is switched off. Moreover QCD possess the property of *asymptotic freedom* i.e the effective coupling goes to zero logarithmically as $Q^2 \to \infty$.

Hence we would expect that as $Q^2 \to \infty$ the term $a_0^{gluons}(Q^2)$ should vanish implying a return to the simple parton model result. But - - - the anomalous gluon contribution is *really* anomalous!

It can be shown that the first moment $\Delta G(Q^2)$ tends to infinity logarithmically as $Q^2 \to \infty$, thus exactly cancelling the decrease in $\alpha(Q^2)$ and the gluonic term survives!

3˙5. *A note on angular momentum sum rules*. – The "spin crisis" was signalled via the failure of

$$S_z^{quarks} = J_z = 1/2\,. \tag{64}$$

which is an intuitive statement that the angular momentum of the nucleon should be made up of the angular momentum of its constituents. This is an example of an angular momentum sum rule, and it seems obviously true. However, such relations in a relativistic theory are very subtle.

Angular momentum sum rules require explicit expressions for the matrix elements of the angular momentum operators and obtaining these is non-trivial. Indeed, for some decades it was believed that one could *not* have an angular momentum sum rule for a *transversely* polarized nucleon, but this was recently shown to be incorrect. [14].

3˙6. *Is the spin crisis really resolved??*. – The key point concerning the axial anomaly is that what is measured, a_0, is **not** equal to $\Delta\Sigma$. Instead one has

$$a_0 = \Delta\Sigma - 3\,\frac{\alpha_s}{2\pi}\,\Delta G \tag{65}$$

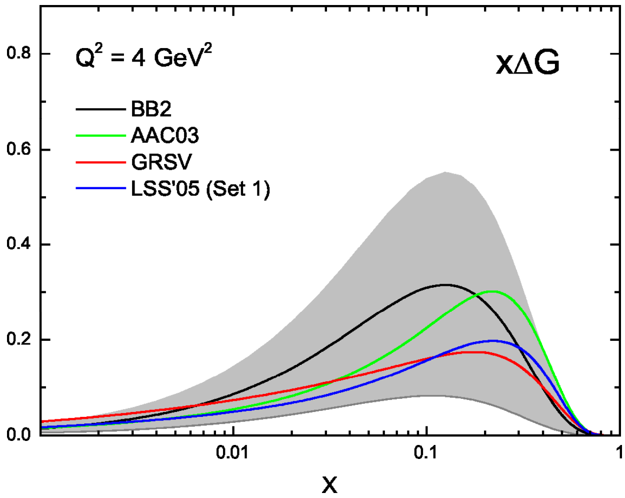

Fig. 6. – $x\,\Delta G(x)$ extracted rom the world DIS data by various groups

so we could have a "big" $\Delta\Sigma$ and a "small" a_0, but this requires a large value of ΔG.

You will see in the lectures of Saito that the measured ΔG seems to be much too small! Presently the best value for a_0 is $a_0 \simeq 0.33$. If we want, say, $\Delta\Sigma \simeq 0.6$ we need $\Delta G \simeq 1.7$ at $Q^2 = 1(GeV/c)^2$. But the latest value for ΔG, obtained from an analysis of the world data on polarized DIS [15] (see fig. 6) is $\Delta G = 0.29 \pm 0.32$, which, even with the large error, looks much too small.

It could be argued that the role of $\Delta G(x)$ in DIS is very indirect. Its main influence is in the evolution with Q^2, and given the limited range of Q^2 available in polarized DIS experiments it is not very well determined. However there are other, more direct ways, of measuring ΔG and they support the conclusion that it is rather small, much too small to resolve the spin crisis. The "golden" method is in the reaction

$$\overleftarrow{\mu} + \overrightarrow{p} \to \mu + \text{two charmed particles}$$

with the charmed particles roughly back to back. Since we assume essentially zero charm content in the nucleon the only possible mechanism is the one shown in fig. 7. Unfortunately, attempting to detect two charmed mesons turns out to be hopeless, from a statistics point of view, so the ΔG extraction is based mainly on detecting a single, charmed meson with large transverse momentum.The results are consistent with a vanishingly small ΔG.

4. – Field theoretic generalization of the parton model

In a field theory whose elementary fields are quarks and gluons i.e. in QCD, one can split the diagram for the hadronic tensor $W^{\mu\nu}$ into a hard part $H^{\mu\nu}$, where the hard

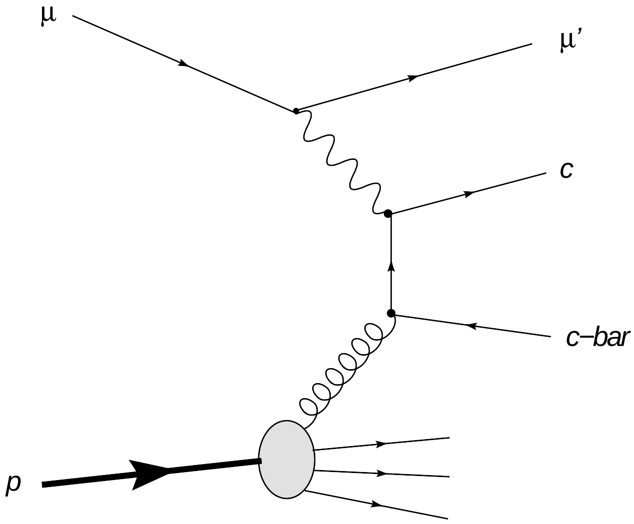

Fig. 7. – Feynman diagram responsible for producing back-to-back charm particles

photon interacts with a quark and a soft part Φ, where the nucleon emits the quark, as shown in fig. 8. Note that this is not a Feynman diagram for an *amplitude*. It represents something like a cross-section, and is really the imaginary part of the forward Compton scattering amplitude for $\gamma + P \to \gamma + P$ with the photon polarization tensors removed. The "blobs" H and Φ are *cut* diagrams and contain on-shell particles as intermediate states. For a more pedagogical explanation of all this, see Chapter 11 of [16].

The blob H, involving hard interactions, can be treated in perturbation theory and

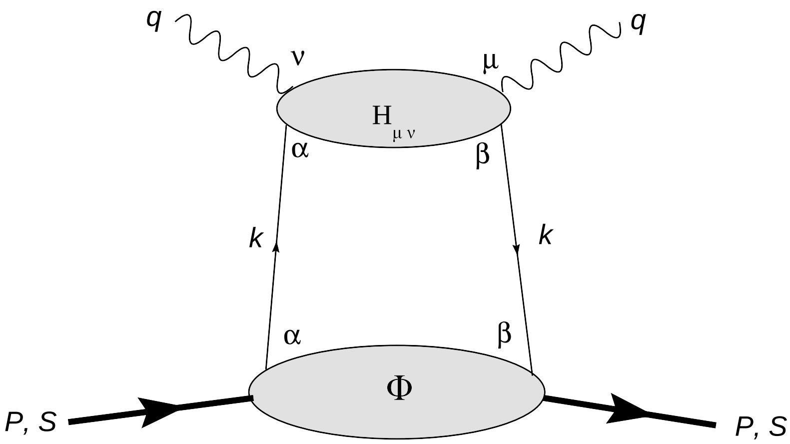

Fig. 8. – QCD generalization of parton model

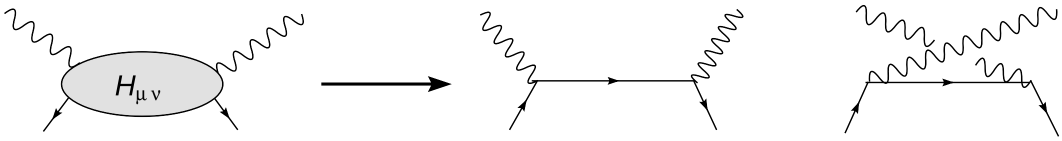

Fig. 9. – Born term expression for the hard part

the leading terms are just the Born terms shown in fig. 9

The blob Φ, involving soft interactions, known as the *quark-quark correlator*, cannot be evaluated explicitly, but its mathematical expression is

$$\Phi_{\alpha\beta}(P,S;k) = \int \frac{d^4z}{(2\pi)^4}\, e^{ik.z}\, \langle P,S|\bar{\psi}_\beta(0)\,\psi_\alpha(z)|P,S\rangle \tag{66}$$

where the quark fields are interacting fields. Note that flavour and the quark charge have been ignored —- they are trivially reinstated at the end —- and that we work with $m_q = 0$.

In terms of these $W^{\mu\nu}$ is given by

$$W^{\mu\nu} = \frac{1}{2\pi} \int \frac{d^4k}{(2\pi)^4}\, H^{\mu\nu}_{\beta\alpha}\, \Phi_{\alpha\beta}(P,S;k) \tag{67}$$

where α, β are Dirac indices.

Note that to regain the results of the simple parton model one approximates H by its Born terms and treats the quark fields in Φ as free fields. Note too that in this language the axial anomaly emerges from the diagrams in fig. 10., in which the right hand diagram involves the *gluon-gluon correlator* Φ_G and the horizontal quark lines are on-shell. In the Bjorken limit the top quark line effectively contracts to a point yielding the anomaly triangle.

4·1. *QCD corrections and evolution.* – Going beyond the parton model means including various QCD corrections. For example in fig. 11 we show the Born term for H and the simplest correction terms, a vertex correction and a diagram where a gluon is radiated from the active quark before it interacts with the photon.

Unfortunately these correction terms are infinite. There are two kinds of infinity:

i) the usual ultra-violet type divergences which have to be eliminated by renormalization

ii) collinear divergences which occur because of the masslessness of the quarks and which are removed by a process known as factorization.

In this the reaction is factorized (separated) into a hard and soft part and the infinity is absorbed into the soft part which in any case cannot be calculated and has to be

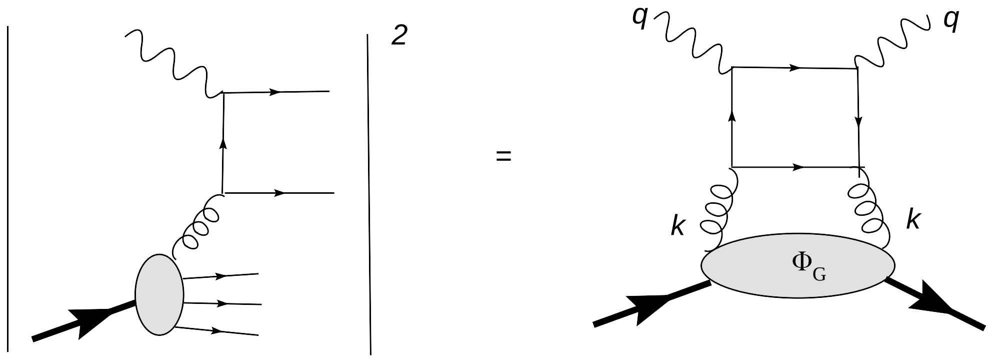

Fig. 10. – QCD diagram leading to the anomaly contribution

parametrized and studied experimentally.

The point at which this separation is made is referred to as the *factorization scale* μ^2. Schematically , one finds terms of the form $\alpha_s\, ln\, \frac{Q^2}{m_q^2}$ which one splits as follows

$$\alpha_s\, ln\, \frac{Q^2}{m_q^2} = \alpha_s\, ln\, \frac{Q^2}{\mu^2} + \alpha_s\, ln\, \frac{\mu^2}{m_q^2} \tag{68}$$

and one then absorbs the first term on the right hand side into the hard part and the

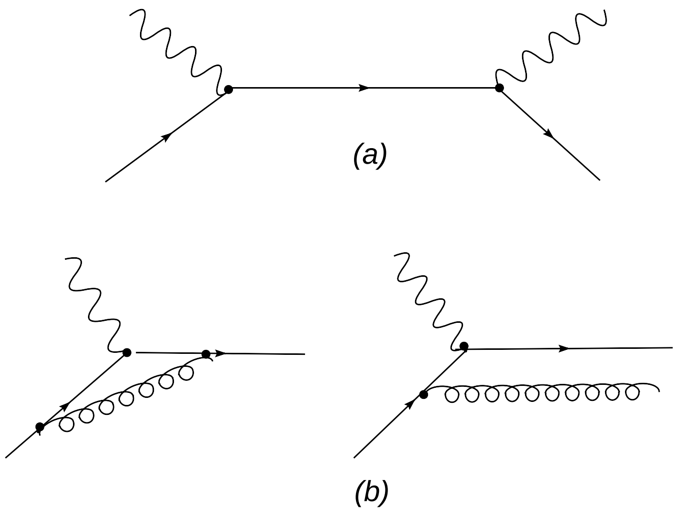

Fig. 11. – Example of QCD correction terms (b), to the Born approximation (a)

second into the soft part. μ^2 is an arbitrary number, like the renormalization scale, and, in an exact calculation, physical results cannot depend on it. However it does mean that what we call the parton density has an extra label μ^2 specifying our choice. Moreover, since we never calculate to all orders in perturbation theory, it can make a difference what value we choose. It turns out that an optimal choice is $\mu^2 = Q^2$, so the parton densities now depend on both x and Q^2 i.e. we have $q(x, Q^2)$ and $\Delta q(x, Q^2)$, and perfect Bjorken scaling is broken. But the variation with Q^2 is gentle (logarithmic), and can be calculated via what are called the *evolution equations* which will be discussed later.

It turns out to be crucial in handling these divergences to use the technique of *dimensional regularization*, which is straightforward in the unpolarized case, but which runs into a snag in the polarized case. The problem is that the generalization of γ_5 in more than 4-dimensions is ambiguous. In 4-dimensions we have

$$\{\gamma^\mu\,,\,\gamma_5\} = 0 \qquad \mu = 0, 1, 2, 3 \tag{69}$$

If we try

$$\{\gamma^n\,,\,\gamma_5\} = 0 \qquad n = 4, 5, \ldots \tag{70}$$

it leads to a contradiction when using

$$Tr[ABC - - - X] = Tr[XABC - - -] \tag{71}$$

There is also a problem with the generalization of $\varepsilon^{\mu\nu\rho\sigma}$. 't Hooft and Veltman [17] and Breitenlohner and Maison [18] suggested using

$$\{\gamma^\mu\,,\,\gamma_5\} = 0 \qquad \mu = 0, 1, 2, 3 \tag{72}$$

$$[\gamma^n\,,\,\gamma_5] = 0 \qquad n = 4, 5, \ldots \tag{73}$$

This gives rise to the $\overline{MS} - HVBM$ *renormalization scheme*, which, however, has a problem. The third component of the isovector axial current $J^3_{\mu_5}$ is NOT conserved, implying that a_3 depends on Q^2. It turns out that this feature is linked to how the factorization between hard and soft parts is implemented and can be remedied.

At present there are three schemes in use, all of them modified versions of $\overline{MS} - HVBM$:

i) The Vogelsang, Mertig, van Neerven scheme [19, 20] $\overline{MS} - MNV$. Here $J^3_{\mu_5}$ is conserved i.e. a_3 is independent of Q^2.

ii)The *AB* scheme of Ball, Forte and Ridolfi [21], which, in addition, has the first moment

$$\Delta\Sigma = \int_0^1 dx\, \Delta\Sigma(x, Q^2) \tag{74}$$

independent of Q^2.

iii) The JET scheme of Carlitz, Collins and Mueller [12], Anselmino. Efremov and Leader [1] and Teryaev and Müller [22], and which is identical to the Chiral Invariant scheme of Cheng [23]. In this scheme a_3 and a_8 are independent of Q^2 as is $\Delta\Sigma$, but it can be argued that the JET scheme is superior to the others in that *all* hard effects are included in H.

Of course if one could work to all orders in perturbation theory it would make no difference which scheme one used, but given that we work to leading order (LO), next to leading order (NLO), and in some cases to NNLO, the choice of scheme can be of importance.

4˙2. *Structure of* $G_1(x, Q^2)$ *at and beyond leading order*. – For the polarized densities the evolution equations are

$$\begin{aligned} \frac{d}{d\,lnQ^2}\,\Delta q(x,Q^2) = \frac{\alpha_s(Q^2}{2\pi} \int_x^1 \frac{dy}{y} \{ & \Delta P_{qq}(x/y)\,\Delta q(y,Q^2) \\ & + \Delta P_{qG}(x/y)\,\Delta G(y,Q^2)\} \end{aligned} \tag{75}$$

$$\begin{aligned} \frac{d}{d\,lnQ^2}\,\Delta G(x,Q^2) = \frac{\alpha_s(Q^2}{2\pi} \int_x^1 \frac{dy}{y} \{ & \Delta P_{Gq}(x/y)\,\Delta q(y,Q^2) \\ & + \Delta P_{GG}(x/y)\,\Delta G(y,Q^2)\} \end{aligned} \tag{76}$$

The ΔP are the polarized *splitting functions* and are calculated perturbatively

$$\Delta P(x) = \Delta P^{(0)}(x) + \frac{\alpha_s}{2\pi}\,\Delta P^{(1)}(x) \tag{77}$$

where the superscripts (0) and (1) refer to LO and NLO contributions. For details about these the reader is referred to Vogelsang [19].

Note that in LO flavour combinations like $q_f - q_{f'}$ e.g. $u(x) - d(x)$ and valence combinations like $q_f - \bar{q}_f$ e.g. $u(x) - \bar{u}(x)$ are *non-singlet* and evolve in the same way, without the ΔG term in eq. (75). (There is no splitting in LO from a q to a $\bar{q}$, nor from say a u to a d.) However, in NLO flavour non-singlets like $u(x) - d(x)$ and charge-conjugation non-singlets like $u(x) - \bar{u}(x)$ evolve differently. The origin of this difference

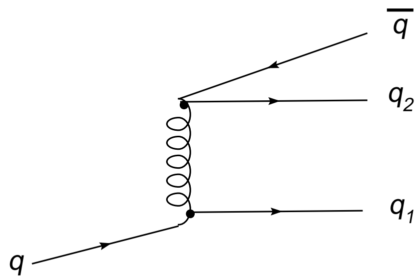

Fig. 12. – NLO amplitude for $q \to \bar{q}$ transition

can be seen in figs. 12 and 13. Fig. 12 shows an NLO amplitude for a quark to split into a $\bar{q}$.

Fig. 13 shows two possible contributions to $\Delta P_{q\bar{q}}$ from taking the modulus squared of this amplitude.

In (a) the contribution is pure flavour singlet and involves only gluon exchange, whereas in (b) the contribution is non-singlet. However, if we try to do something similar for a flavour changing splitting function e.g. ΔP_{du} we find that we cannot construct the non-singlet diagram.

The expression for $g_1(x, Q^2)$ now becomes

$$
\begin{aligned}
g_1(x, Q^2) = \frac{1}{2} \sum_{flavours} e_q^2 \Big\{ \Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2) \\
+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{ \Delta C_q(x/y) \, [\Delta q(y, Q^2) + \Delta\bar{q}(y, Q^2)] \\
+ \Delta C_G(x/y) \, \Delta G(y, Q^2) \} \Big\} \qquad (78)
\end{aligned}
$$

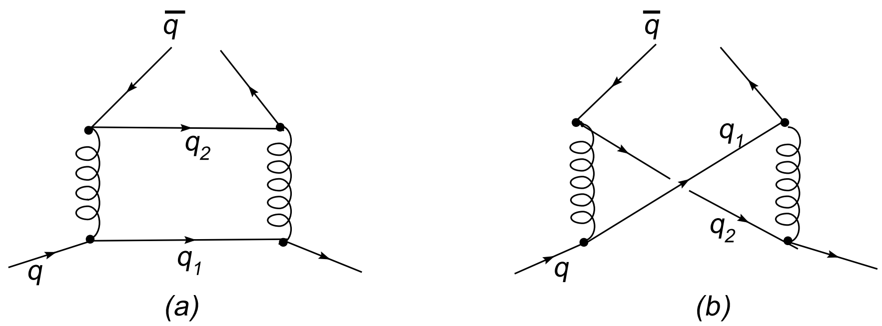

Fig. 13. – NLO contributions to the splitting function for a $q \to \bar{q}$ transition

where ΔC_G and ΔC_q are Wilson coefficients evaluated from the hard part calculated beyond the Born approximation.

Note that very often the evolution equations are written using the *convolution notation*, for example,

$$\Delta C_q \otimes \Delta q \equiv \int_x^1 \frac{dy}{y}\, \Delta C_q(x/y)\, \Delta q(y) \tag{79}$$

5. – The polarized strange quark density: attempts to measure $\Delta s(x)$

Although the srangeness content of the nucleon is small, it has played a major role in provoking puzzles and controversies in our understanding of the internal structure of the nucleon, particularly as concerns the spin structure. Recall that it was a misjudgement of the significance of strangeness in the Ellis-Jaffe result that was behind the original excitement generated by the famous EMC experiment in 1988. And as we shall now see there is still some mystery surrounding the polarized strange density.

There are two possibilities for measuring $\Delta s(x)$, via polarized DIS or via polarized semi-inclusive DIS (SIDIS).

Recall that DIS only depends on $\Delta q(x) + \Delta\bar{q}(x)$. So we can obtain information on $\Delta s(x) + \Delta\bar{s}(x)$.

In SIDIS we could, in principle obtain $\Delta s(x)$ and $\Delta\bar{s}(x)$ separately, but that is for the future!

5˙1. *Results from polarized DIS*. – Aside from one small issue there is general agreement between several analyzes: see fig. 14.

What causes the disagreement at moderate to large x? Surprisingly—-*positivity* i.e. the requirement that

$$|\Delta s(x)| \le s(x) \tag{80}$$

As shown in fig. 15 the data seem to want a large negative $\Delta s(x)$ at moderate values of x.

So there is a clash with positivity and the result is that the shape of $\Delta s(x)$ is sensitive to the input *unpolarized* density. In the figure the polarized analyses BB2 [24], AAC03 [25] and GRSV [26] utilized the unpolarized strangeness density of GRV98 [27], whereas LSS05(Set 1) [15] used the unpolarized strangeness density of MRST'02 [28]. It is seen that LSS05 is incompatible with the unpolarized GRV density.

5˙2. *Results from SIDIS*. – Before looking at results consider the following constraint [29] on the first moment

$$\delta_s \equiv [\Delta s + \Delta\bar{s}] \tag{81}$$

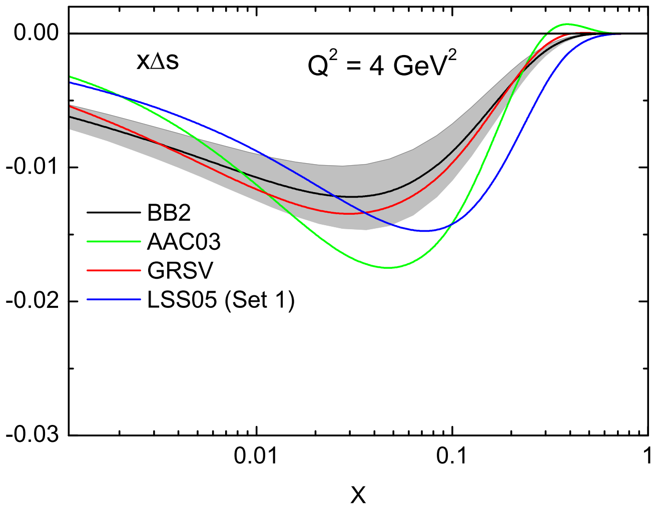

Fig. 14. – Polarized strange quark density from various analyzes of world data on DIS

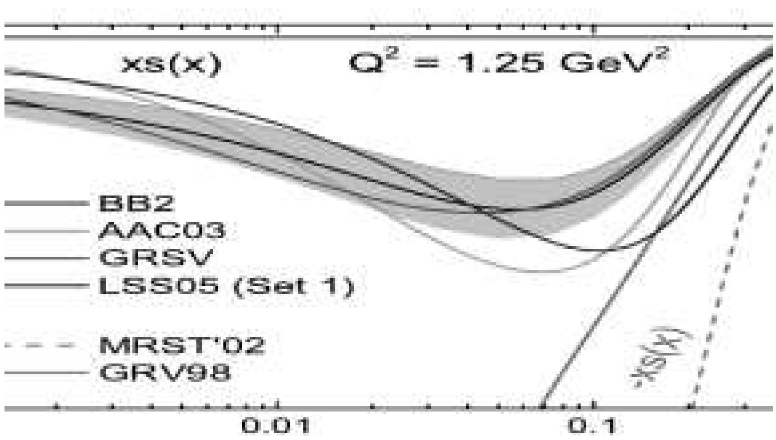

Fig. 15. – Role of positivity in influencing $\Delta q(x)$ at moderate values of x

We can rewrite the expression for Γ_1^p as

$$\Gamma_1^p(Q^2) = \frac{1}{6}\big[\frac{1}{2}a_3 + \frac{5}{6}a_8 + 2\delta_s(Q^2)\big] \tag{82}$$

or

$$a_8 = \frac{6}{5}\big[6\Gamma_1^p(Q^2) - \frac{1}{2}a_3 - 2\delta_s(Q^2)\big] \tag{83}$$

We know a_3 very accurately. Using the measured values of $\Gamma_1^p(Q^2)$ we show that $\delta_s(Q^2) \geq 0$ implies an unacceptable value for a_8.

We have to decide what value to use for $\Gamma_1^p(Q^2)$, since the result depends on the extrapolation to $x = 0$. We take two extremes:

(i) Assume perturbative QCD holds at small x as done by SLAC experiment E155 [30] etc. This yields

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004 \pm 0.007 \tag{84}$$

(ii) Assume Regge behaviour at small x as utilized by SLAC experiment E143 [31] etc. This gives

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003 \pm 0.009 \tag{85}$$

Results: If δ_s is *positive* we find:

(i) $a_8 \leq 0.089 \pm 0.058$

(ii) $a_8 \leq 0.197 \pm 0.068$

Now to the best of our knowledge hyperon β-decay is adequately described by $SU(3)_F$ and this leads to $a_8 = 0.585 \pm 0.025$

Thus $\delta_s(Q^2) \geq 0$ implies a dramatic breaking of $SU(3)_F$, and we conclude that it is *almost* impossible to have $\delta_s(Q^2) \geq 0$.

Now HERMES has extracted $\Delta s(x) + \Delta\bar{s}(x)$ from a study of SIDIS [32]. The results are shown in fig. 16.

Within errors the results are consistent with zero, and HERMES quote

$$\delta_s(Q^2 = 2.5) = 0.028 \pm 0.033 \pm 0.009 \tag{86}$$

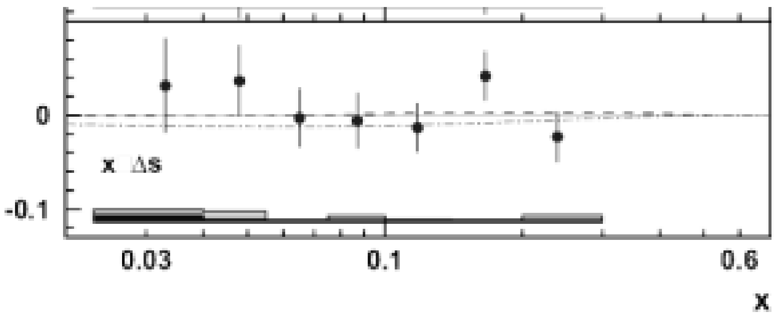

Fig. 16. – $x\,[\Delta s(x) + \Delta\bar{s}(x)]$ from HERMES SIDIS analysis

The previous discussion suggests that the central value *cannot* be the true value unless we have totally failed to understand the connection between DIS and SIDIS . If the latter is not the case, how can we understand the HERMES results?

I think it is important to remember that HERMES uses a LO method based on so-called *purities*. I suspect that such an approach is unreliable at the values of Q^2 involved, and that the errors on the purities are somewhat underestimated in their analysis. So I strongly believe that this new 'strange quark crisis' will prove to be illusory.

6. – A last word on the "spin crisis"

We have seen that the hope that a large polarized gluon density could resolve the "spin crisis" is probably no longer tenable, given that recent experiments seem to be indicating quite a small value for ΔG. If that is so, how are we to resolve the crisis? The answer is, in principle, quite straightforward. In our collinear parton model we neglected transverse motion of the partons, and this transverse momentum can generate orbital angular momentum with a component in the direction of motion of the nucleon. It is a simple exercise to show that an acceptable magnitude of transverse momentum could yield enough L_z to satisfy the longitudinal angular momentum sum rule. Against this explanation is the intuitive, but probably incorrect, argument that in quark models of hadrons the nucleon appears as an s-wave ground state i.e. with zero orbital angular momentum.

Acknowledgements: Almost all of the work reported here was carried out in collaboration with A. V. Siderov (Bogoliubov Theoretical Laboratory, Dubna) and D. B. Stamenov (Institute for Nuclear Research and Nuclear Energy, Sofia).

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