Intrinsic parton motion and the longitudinal spin asymmetry A_{LL} in high energy $pp \to \pi X$

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The longitudinal double spin asymmetry A_{LL} in the reaction $pp \to \pi X$ has been measured at RHIC with extremely interesting consequences. If the gluon polarization in a proton were as big as needed to resolve the famous "spin crisis" then A_{LL} would be large and positive. Early RHIC results suggested that A_{LL} might even be negative, which is impossible in the simple collinear parton model. Later results indicate very small positive values. Recently, for the first time, we derived expressions for the general partonic structure for all hadron spin asymmetries with inclusion of all transverse motion of the partons in a hadron and of the hadrons in a fragmenting parton. Besides the standard soft functions that are present in the collinear treatment, several new spin and $\boldsymbol{k}_\perp$ dependent soft functions appear and contribute to the cross sections and to spin asymmetries, both transverse and longitudinal. We examine the influence of $\boldsymbol{k}_\perp$ and of the new terms on A_{LL}, and in particular whether they could alter the conclusion that the gluon polarization is very small. It turns out that the contribution from these effects is essentially negligible.

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I. INTRODUCTION

In the QCD Parton Model there is a significant difference between longitudinal and transverse spin asymmetries. Longitudinal double spin asymmetries can be generated directly in terms of the longitudinal polarized parton densities, even in the simplest version of the model with collinear kinematics, and can be sizeable; longitudinal single spin asymmetries are forbidden by parity invariance and one needs high energy parity violating effects to observe them. On the contrary, transverse double spin asymmetries in high energy hadronic inclusive collisions are expected to be very small at mid rapidity due to the non transverse nature of the gluonic contributions [1], and transverse single spin asymmetries (SSA) are allowed by parity invariance, but essentially vanish in the collinear approximation at leading twist.

Transverse SSA can be generated in two ways, either in a field-theoretic approach by including higher twist quark-quark-gluon correlators [2, 3], or in the parton model by including the transverse motion of the partons [4, 5] and introducing new soft functions, which violate naive time reversal invariance. The two approaches might be correlated [6]. These new functions induce single spin dependences in the partonic distributions and in the fragmentation processes according to four new mechanisms: the Sivers effect [4] whereby in a transversely polarized nucleon the number density of quarks with momentum $\boldsymbol{k}$ can depend upon the azimuthal angle between $\boldsymbol{k}$ and the spin vector of the nucleon; the Boer-Mulders effect [7, 8] which allows partons to be transversely polarized in an unpolarized nucleon; the Collins effect [9] whereby the number density of unpolarized hadrons with momentum $\boldsymbol{k}$ produced in the fragmentation of a transversely polarized quark can depend on the azimuthal angle between $\boldsymbol{k}$ and the spin vector of the quark; a mechanism whereby an unpolarized quark can fragment into a polarized hadron [7, 10, 11]. We have shown that in reactions in which an unpolarized hadron is produced the transverse single spin asymmetries are dominated by the Sivers mechanism, because phase cancelations greatly reduce the contributions of the other mechanisms [12].

What is perhaps surprising is that these mechanisms, invented in order to produce transverse asymmetries, also contribute to the total cross-section and to the longitudinal spin asymmetries. It has been shown that their effect in total cross-sections is negligible [13, 14], but given that A_{LL} is so small it is important to check whether they can have a significant influence upon its value, in particular whether they could produce a small value of A_{LL} in conjunction with a large gluon polarization. We find that the new contributions can indeed be negative, thus reducing the value of A_{LL}, but that their magnitude is much too small to be relevant. Thus the conclusion that the small positive value of A_{LL} implies a very small gluon polarization is unaltered.

The plan of the paper is the following. In Section II we briefly recall the formalism used for our calculation, which includes the full non-collinear kinematics of the scattering process. In Section III we present the "kernels" for the calculation of each partonic contribution to the polarized cross-sections. In Section IV we show and discuss our phenomenological results for the longitudinal double spin asymmetry in inclusive neutral pion production at RHIC. Finally, in Section V we draw our conclusions.

II. FORMALISM

Here we simply sketch the main aspects of the formalism; for details of the approach we refer to [14]. The longitudinal double spin asymmetry A_{LL} for the reaction $pp \to \pi X$ is defined as

$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} = \frac{d\sigma^{++} - d\sigma^{+-}}{2d\sigma^{unp}}\,, \tag{1}$$

where the labels refer to the helicities of the protons.

The general expression for the differential cross-sections for the polarized hadronic process $(A, S_A)+(B, S_B) \to C+X$ is given by

$$\begin{aligned}\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} &= \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c) \\ &\times J(\boldsymbol{k}_{\perp C})\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \\ &\times \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,\end{aligned} \tag{2}$$

which involves a (factorized) convolution of all possible hard elementary QCD processes, $ab \to cd$, with soft partonic polarized distribution and fragmentation functions. In Eq. (2) $\hat{s}, \hat{t}$ and $\hat{u}$ are the Mandelstam variables for the partonic reactions and the detailed connection between the hadronic and the partonic kinematical variables is given in full in Appendix A of [14].

Let us simply recall here, for a better understanding, the physical meaning of the different factors in Eq. (2):

- $\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}$ is the helicity density matrix of parton a inside the polarized hadron A, with spin state S_A; it describes the parton polarization. $\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a})$ is the number density (or distribution) of unpolarized partons a inside the polarized hadron A, S_A: each parton carries a light-cone momentum fraction x_a and a transverse momentum $\boldsymbol{k}_{\perp a}$. Similarly for parton b inside hadron B with spin S_B.

- The polarized cross-sections for the elementary partonic process $(a, s_a) + (b, s_b) \to (c, s_c) + d$ are expressed in terms of products of the helicity amplitudes $\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$.

- The factor $\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z, \boldsymbol{k}_{\perp C})$ describes, again in the helicity basis, the fragmentation process $c \to C+X$, according to which a polarized parton c fragments into an unpolarized hadron C carrying a light-cone momentum fraction z and a transverse momentum $\boldsymbol{k}_{\perp C}$.

- $J(\boldsymbol{k}_{\perp C})$ is a kinematical factor, numerically very close to 1 for RHIC kinematics. All details can be found in Ref. [14]. Throughout the paper, we work in the AB c.m. frame, assuming that hadron A moves along the positive Z_{cm}-axis and hadron C is produced in the $(XZ)_{cm}$ plane, with $(p_C)_{X_{cm}} > 0$.

Eq. (2) is written in a factorized form, separating the soft, long distance from the hard, short distance contributions. The hard part is computable in perturbative QCD, while information on the soft one has to be extracted from other experiments or modeled. As already mentioned and discussed in Ref. [12], such a factorization with noncollinear kinematics has never been formally proven. Indeed, studies of factorization [15–18], comparing semi-inclusive deep inelastic scattering (SIDIS) and Drell-Yan reactions have indicated unexpected modifications of simple factorization, and the situation for single inclusive particle production in hadron–hadron collisions is not yet resolved. Thus, our approach can only be considered as the natural extension of the collinear case and a reasonable phenomenological model. Of course, the perturbative calculation of the hard part is only reliable if the hard scale – in this case the square of the transverse momentum of the final hadron, p_T^2 – is large enough. It turns out that the data on unpolarized cross-sections demand [13] an average value of $k_\perp^2 \equiv |\boldsymbol{k}_\perp|^2 \simeq 0.64$ (GeV$/c$)^2 for the intrinsic transverse momentum of the parton distributions. We shall study how the contributions to A_{LL} depend on the value of $\langle k_\perp^2\rangle$.

III. KERNELS

As we can see from Eq. (2), the computation of the cross-section corresponding to any polarized hadronic process $(A,S_A)+(B,S_B)\to C+X$ requires the evaluation and integration, for each elementary process $a+b\to c+d$, of the general kernel

$$\begin{aligned}\Sigma(S_A,S_B)^{ab\to cd} &= \sum_{\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\,\hat f_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\,\hat f_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})\\ &\times \hat M_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\hat M^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\,\hat D^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,. \end{aligned} \tag{3}$$

While the hadronic process $(A,S_A)+(B,S_B)\to C+X$ takes place, according to our choice, in the $(XZ)_{cm}$ plane, all the elementary processes involved, $A(B)\to a(b)+X$, $ab\to cd$ and $c\to C+X$ do not, since all parton and hadron momenta, $\boldsymbol{p}_a$, $\boldsymbol{p}_b$, $\boldsymbol{p}_C$ have transverse components $\boldsymbol{k}_{\perp a}$, $\boldsymbol{k}_{\perp b}$, $\boldsymbol{k}_{\perp C}$. This "out of $(XZ)_{cm}$ plane" geometry induces phases in the fragmentation process, in the distribution functions and in the elementary interactions, which have to be taken into account. Thus, the independent helicity amplitudes for the elementary pQCD processes $ab\to cd$, with massless partons, can be written as [14]

$$\hat M_{+,+;+,+}\equiv \hat M^0_1\,e^{i\varphi_1}\qquad \hat M_{-,+;-,+}\equiv \hat M^0_2\,e^{i\varphi_2}\qquad \hat M_{-,+;+,-}\equiv \hat M^0_3\,e^{i\varphi_3}, \tag{4}$$

where the amplitudes $\hat M^0_{1,2,3}$ are the real planar amplitudes defined in the partonic $ab\to cd$ c.m. frame,

$$\hat M^0_1\equiv \hat M^0_{+,+;+,+}=\hat M^0_{-,-;-,-}\qquad \hat M^0_2\equiv \hat M^0_{-,+;-,+}=\hat M^0_{+,-;+,-}\qquad \hat M^0_3\equiv \hat M^0_{-,+;+,-}=\hat M^0_{+,-;-,+}, \tag{5}$$

as required by parity invariance. The phases $\varphi_{1,2,3}$ are complicated functions of the polar and azimuthal angles of the transverse momenta, $\boldsymbol{k}_{\perp a},\boldsymbol{k}_{\perp b}$ and $\boldsymbol{k}_{\perp C}$, and their explicit expressions can be found in Ref. [14]. The relations

$$\hat M_{-,-;-,-}=\hat M^*_{+,+;+,+}\qquad \hat M_{+,-;+,-}=\hat M^*_{-,+;-,+}\qquad \hat M_{+,-;-,+}=\hat M^*_{-,+;+,-}, \tag{6}$$

follow from Eqs. (4), (5) and from the fact that the phases φ_i change sign by helicity inversion [14]. Note that the $+$ and $-$ subscripts refer to $(+1/2)$ and $(-1/2)$ helicities for quarks, and to $(+1)$ and (-1) helicities for gluons. There are eight elementary contributions $ab\to cd$ which we have to consider separately

$$\begin{aligned} &q_aq_b\to q_cq_d\,,\quad g_ag_b\to g_cg_d\,,\\ &qg\to qg\,,\quad gq\to gq\,,\\ &qg\to gq\,,\quad gq\to qg\,,\\ &g_ag_b\to q\bar q\,,\quad q\bar q\to g_cg_d\,, \end{aligned} \tag{7}$$

where q can in general be either a quark or an antiquark. The subscripts a,b,c,d for quarks, when necessary, identify the flavour (only in processes where different flavours can be present); for gluons, these labels identify the corresponding hadron ($a\to A$, $b\to B$, $c\to C$). By performing the explicit sums in Eq. (3), we obtain the kernels for each of the elementary processes. Note that the new aspect of our calculation is the appearance of the phases which is a reflection of the noncollinear kinematics.

The computation of the denominator/numerator of A_{LL} in Eq. (1) requires the evaluation of the kernels $[\Sigma(+,+)\pm\Sigma(+,-)]$ respectively. The expressions for the sums of kernels, which are relevant for the unpolarized cross-section, are given in [14]. Here we give in detail the expressions for the differences. They are calculated from the general kernel given in Eq. (3). In the following certain terms are underlined: these are terms which vanish after integration over the angles of the momenta $\boldsymbol{k}_{\perp a}$, $\boldsymbol{k}_{\perp b}$, $\boldsymbol{k}_{\perp C}$ in Eq. (2), as required by parity invariance; we shall further comment on that at the end of this Section. ϕ^H_C is the azimuthal angle of the hadron C in the parton c helicity frame and its expression in terms of the angles of $\boldsymbol{k}_{\perp C}$ is given in Appendix A of Ref. [14]. Notice that all angular dependences of the kernels are explicitly extracted and the parton distribution and fragmentation functions only depend on the magnitudes of the transverse momentum vectors.

- $q_aq_b\to q_cq_d$ contribution

$$\begin{aligned}&[\Sigma(+,+)-\Sigma(+,-)]^{q_aq_b\to q_cq_d} =\\ &\quad \Delta\hat f^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat f^b_{s_z/+}(x_b,k_{\perp b})\,\left[|\hat M^0_1|^2-|\hat M^0_2|^2-|\hat M^0_3|^2\right]\,\hat D_{C/c}(z,k_{\perp C})\\ &+\Bigg[\Delta\hat f^a_{s_x/+}(x_a,k_{\perp a})\,\Delta\hat f^b_{s_x/+}(x_b,k_{\perp b})\,\cos(\varphi_3-\varphi_2)\end{aligned}$$

$$+\underline{\Delta\hat{f}^a_{s_y/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_x/+}(x_b,k_{\perp b})\,\sin(\varphi_3-\varphi_2)}\Big]\,(2\,\hat{M}^0_2\,\hat{M}^0_3)\,\hat{D}_{C/c}(z,k_{\perp C})$$
$$-\;\underline{\hat{f}_{a/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_x/+}(x_b,k_{\perp b})\,\hat{M}^0_1\,\hat{M}^0_3\,\sin(\varphi_1-\varphi_3+\phi^H_C)\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})}\,. \tag{8}$$

Notice that we have used the relations $\Delta\hat{f}^a_{s_y/+}(x_a,k_{\perp a})=\Delta\hat{f}^a_{s_y/A}(x_a,k_{\perp a})$ and $\hat{f}_{a/+}(x_a,k_{\perp a})=\hat{f}_{a/A}(x_a,k_{\perp a})$, see Appendix B of Ref. [14]. The channels $q\bar{q}\to q\bar{q}$ etc. are formally identical to $qq\to qq$ with amplitudes defined properly in Ref. [14].

- $g_a g_b \to g_c g_d$ contribution

$$\begin{aligned}
&[\Sigma(+,+)-\Sigma(+,-)]^{g_a g_b\to g_c g_d} = \\
&\quad \Delta\hat{f}^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_z/+}(x_b,k_{\perp b})\,\Big[|\hat{M}^0_1|^2-|\hat{M}^0_2|^2-|\hat{M}^0_3|^2\Big]\,\hat{D}_{C/g}(z,k_{\perp C}) \\
&+\;\Big[\Delta\hat{f}^a_{\mathcal{T}_2/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{\mathcal{T}_2/+}(x_b,k_{\perp b})\,\cos(\varphi_3-\varphi_2) \\
&\quad +\underline{\Delta\hat{f}^a_{\mathcal{T}_1/A}(x_a,k_{\perp a})\Delta\hat{f}^b_{\mathcal{T}_2/+}(x_b,k_{\perp b})\,\sin(\varphi_3-\varphi_2)}\Big]\,(2\,\hat{M}^0_2\,\hat{M}^0_3)\,\hat{D}_{C/g}(z,k_{\perp C}) \\
&+\;\underline{\hat{f}_{a/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{\mathcal{T}_2/+}(x_b,k_{\perp b})\,\hat{M}^0_1\,\hat{M}^0_3\,\sin(\varphi_1-\varphi_3+2\phi^H_C)\,\Delta^N\hat{D}_{C/\mathcal{T}^g_1}(z,k_{\perp C})}
\end{aligned} \tag{9}$$

- $q\bar{q}\to gg$ contribution

$$\begin{aligned}
&[\Sigma(+,+)-\Sigma(+,-)]^{q\bar{q}\to gg} = \\
&-\;\Delta\hat{f}^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_z/+}(x_b,k_{\perp b})\,\Big[|\hat{M}^0_2|^2+|\hat{M}^0_3|^2\Big]\,\hat{D}_{C/g}(z,k_{\perp C}) \\
&+\;\Big[\Delta\hat{f}^a_{s_x/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_x/+}(x_b,k_{\perp b})\,\cos(\varphi_3-\varphi_2) \\
&\quad +\underline{\Delta\hat{f}^a_{s_y/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_x/+}(x_b,k_{\perp b})\,\sin(\varphi_3-\varphi_2)}\Big]\,(2\,\hat{M}^0_2\,\hat{M}^0_3)\,\hat{D}_{C/g}(z,k_{\perp C})
\end{aligned} \tag{10}$$

- $g_a g_b\to q\bar{q}/\bar{q}q$ contribution

$$\begin{aligned}
&[\Sigma(+,+)-\Sigma(+,-)]^{g_a g_b\to q\bar{q}/\bar{q}q} = \\
&-\;\Delta\hat{f}^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_z/+}(x_b,k_{\perp b})\,\Big[|\hat{M}^0_2|^2+|\hat{M}^0_3|^2\Big]\,\hat{D}_{C/c}(z,k_{\perp C}) \\
&+\;\Big[\Delta\hat{f}^a_{\mathcal{T}_2/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{\mathcal{T}_2/+}(x_b,k_{\perp b})\,\cos(\varphi_3-\varphi_2) \\
&\quad +\underline{\Delta\hat{f}^a_{\mathcal{T}_1/A}(x_a,k_{\perp a})\Delta\hat{f}^b_{\mathcal{T}_2/+}(x_b,k_{\perp b})\,\sin(\varphi_3-\varphi_2)}\Big]\,(2\,\hat{M}^0_2\,\hat{M}^0_3)\,\hat{D}_{C/c}(z,k_{\perp C})
\end{aligned} \tag{11}$$

- $qg\to qg$ contribution

$$\begin{aligned}
&[\Sigma(+,+)-\Sigma(+,-)]^{qg\to qg} = \\
&\quad \Delta\hat{f}^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_z/+}(x_b,k_{\perp b})\,\Big[|\hat{M}^0_1|^2-|\hat{M}^0_2|^2\Big]\,\hat{D}_{C/c}(z,k_{\perp C})
\end{aligned} \tag{12}$$

- $gq\to qg$ contribution

$$\begin{aligned}
&[\Sigma(+,+)-\Sigma(+,-)]^{gq\to qg} = \\
&\quad \Delta\hat{f}^a_{s_z/+}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_z/+}(x_b,k_{\perp b})\,\Big[|\hat{M}^0_1|^2-|\hat{M}^0_3|^2\Big]\,\hat{D}_{C/c}(z,k_{\perp C}) \\
&-\;\underline{\hat{f}_{a/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_x/+}(x_b,k_{\perp b})\,\hat{M}^0_1\,\hat{M}^0_3\,\sin(\varphi_1-\varphi_3+\phi^H_C)\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})}
\end{aligned} \tag{13}$$

- $qg \to gq$ contribution

$$
\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{qg\to gq} = \\
&\qquad \Delta\hat{f}^{a}_{s_z/+}(x_a, k_{\perp a})\, \Delta\hat{f}^{b}_{s_z/+}(x_b, k_{\perp b}) \left[|\hat{M}^0_1|^2 - |\hat{M}^0_3|^2 \right] \hat{D}_{C/g}(z, k_{\perp C}) \\
&+ \underline{\hat{f}_{a/A}(x_a, k_{\perp a})\, \Delta\hat{f}^{b}_{\mathcal{T}_2/+}(x_b, k_{\perp b})\, \hat{M}^0_1\, \hat{M}^0_3\, \sin(\varphi_1 - \varphi_3 + 2\phi^H_C)\, \Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})}
\end{aligned}
\tag{14}
$$

- $gq \to gq$ contribution

$$
\begin{aligned}
&[\Sigma(+,+) - \Sigma(+,-)]^{gq\to gq} = \\
&\qquad \Delta\hat{f}^{a}_{s_z/+}(x_a, k_{\perp a})\, \Delta\hat{f}^{b}_{s_z/+}(x_b, k_{\perp b}) \left[|\hat{M}^0_1|^2 - |\hat{M}^0_2|^2 \right] \hat{D}_{C/g}(z, k_{\perp C})\,.
\end{aligned}
\tag{15}
$$

The physical content of the above expressions is interesting. First note the complete formal symmetry between the $qq \to qq$ kernel in Eq. (8) and the $gg \to gg$ kernel in Eq. (9). These kernels contain the largest variety of contributions, and the kernels for all the other partonic processes can be formally read off from these by the suppression of certain terms.

In the second line of both expressions, Eq. (8) and Eq. (9), we recognize the product of the $k_\perp$-dependent helicity distributions, $\Delta\hat{f}^{q}_{s_z/+}(x_q, k_{\perp q}) \equiv \Delta q(x_q, k_{\perp q})$ and $\Delta\hat{f}^{g}_{s_z/+}(x_g, k_{\perp g}) \equiv \Delta g(x_g, k_{\perp g})$ for quarks and gluons respectively, and the unpolarized fragmentation function $\hat{D}_{C/c}(z, k_{\perp C})$, with no azimuthal phases. In the third line of Eq. (8) we have two parton distribution functions $\Delta\hat{f}^{q}_{s_x/+}(x, k_\perp)$ referring to quarks transversely polarized, along the x-axis, inside longitudinally polarized nucleons, coupled to the unpolarized fragmentation function. Analogously, in the third line of Eq. (9) we have two parton distribution functions $\Delta\hat{f}^{g}_{\mathcal{T}_2/+}(x, k_\perp)$ which are related to the linear polarization of a gluon inside a longitudinally polarized nucleon. Correspondingly the fourth line of Eq. (8) refers to one quark transversely polarized along the x-axis inside a longitudinally polarized nucleon and the other, $\Delta\hat{f}^{q}_{s_y/A}(x, k_\perp)$, transversely polarized along the y-axis inside an unpolarized nucleon – the latter is the Boer-Mulders function – coupled to the unpolarized fragmentation function. Analogously, in the fourth line of Eq. (9) we have $\Delta\hat{f}^{g}_{\mathcal{T}_2/+}(x, k_\perp)$ and the "Boer-Mulders-like" gluon function, $\Delta\hat{f}^{g}_{\mathcal{T}_1/A}(x, k_\perp)$, referring to a linearly polarized gluon inside an *unpolarized* nucleon. For a more complete explanation of the physical meaning of these functions see Appendix B of [14]. Finally, the last line of Eq. (8) contains the Collins fragmentation function, $\Delta^N\hat{D}_{C/c^\uparrow}(z, k_{\perp C})$, coupled to an unpolarized parton density and a transversely polarized one. In the case of the gluon, in the last line of Eq. (9), there appears a gluonic analogue of the Collins fragmentation function, $\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z, k_{\perp C})$, describing the fragmentation of a linearly polarized gluon into an unpolarized hadron.

Ignoring the underlined terms which vanish upon integration, we see that compared to the standard collinear approach, we have extra contributions involving quarks polarized transversely along their x-axis in a longitudinally polarized nucleon, appearing in Eqs. (8), (10) and contributions involving linearly polarized gluons inside a longitudinally polarized nucleon, appearing in Eqs. (9), (11). Notice that the processes in Eqs. (12)-(15), initiated by quark-gluon elementary scattering, get contributions only from the usual terms, which survive in the collinear case.

The reason why the underlined terms in Eqs. (8)-(15) vanish upon angular integration is the parity invariance of the strong interactions; the demonstration of their vanishing requires a detailed study of the kinematics. One can show, considering the relationship between the angular integration variables appearing in $\boldsymbol{k}_{\perp a}$, $\boldsymbol{k}_{\perp b}$, $\boldsymbol{k}_{\perp C}$ in Eq. (2) and the phase variables $\varphi_{1,2,3}$ and ϕ^H_C [14], that a parity transformation implies $\varphi_i \to -\varphi_i$ $(i = 1, 2, 3)$ and $\phi^H_C \to -\phi^H_C$. Thus the odd sin terms in Eqs. (8)-(15) must vanish if parity is conserved. We have numerically checked that this is indeed the case.

Another simple, but interesting example of such a vanishing can be obtained by considering, within the same formalism, the expression of the kernels for the longitudinal *single spin* asymmetry A_L, which we know *must* vanish in a parity conserving theory. The kernels themselves are *not* zero, but under integration do vanish. This is another very stringent test of the correctness of our formalism. For A_L, for the partonic channel $q_a q_b \to q_c q_d$, we have for the numerator of the longitudinal single spin asymmetry the following expression:

$$
\begin{aligned}
&[\Sigma(+,0) - \Sigma(-,0)]^{q_a q_b\to q_c q_d} = \\
&\qquad \underline{\Delta\hat{f}^{a}_{s_x/+}(x_a, k_{\perp a})\, \Delta\hat{f}^{b}_{s_y/B}(x_b, k_{\perp b})\, (2\hat{M}^0_2\, \hat{M}^0_3)\, \sin(\varphi_3 - \varphi_2)\, \hat{D}_{C/c}(z, k_{\perp C})} \\
&\qquad \underline{-\Delta\hat{f}^{a}_{s_x/+}(x_a, k_{\perp a})\, \hat{f}_{b/B}(x_b, k_{\perp b})\, \sin(\varphi_1 - \varphi_3 + \phi^H_C)\, \hat{M}^0_1\, \hat{M}^0_2\, \Delta^N\hat{D}_{C/c^\uparrow}(z, k_{\perp C})}\,,
\end{aligned}
\tag{16}
$$

and, again, all terms – being odd functions of φ_i and ϕ^H_C – vanish, as they should, upon angular integration.

IV. PHENOMENOLOGY: A_{LL} AT RHIC

The longitudinal double spin asymmetry A_{LL} for inclusive neutral pion production in proton-proton scattering has been measured at RHIC in various runs by the PHENIX and STAR Collaborations. Their first published experimental data [19, 20] showed results for A_{LL} at mid rapidity compatible with large negative values. This was quite puzzling, since A_{LL} is a positive quantity in the collinear parton model, at least at low p_T where it is dominated by $gg \to gg$ elementary scattering processes, see Eq. (9) in which it can be shown that $|\hat{M}_1^0|^2 - |\hat{M}_2^0|^2 - |\hat{M}_3^0|^2 > 0$. More recent and precise data from both collaborations [21, 22] exclude the possibility of a large and negative A_{LL}: in two subsequent RHIC runs, 5 and 6 (results from Run 6 have only been presented as "preliminary" [23, 24]), they confirm and reinforce the statement that A_{LL} is very small and compatible with zero over the whole p_T range they cover, from 1 to 8 GeV/c approximately.

Next to leading order QCD calculations of A_{LL} in a collinear configuration have previously shown that the present data disfavour large positive values for Δg and definitely exclude scenarios where Δg is as large as the unpolarized gluon distribution function, g, at low scale [25]. Instead they are in better agreement with the predictions obtained by assuming $\Delta g = 0$ or even $\Delta g = -g$ at the initial scale [25]. A recent statistical analysis shows that the PHENIX Run 5 data are compatible with both $\Delta g = 0$ and the "standard" GRSV parametrization [26] at 12% and 20% confidence level respectively, while it rules out the $\Delta g = -g$ hypothesis [23]. Instead, its newest update, which includes the preliminary data from PHENIX Run 6, favours the $\Delta g = 0$ scenario over the standard GRSV with 44% confidence level, against 0.25% [24]. (Note that in this Section we have adopted the common, short-hand notation $\Delta f^q_{s_z/+} \equiv \Delta q$ and $\Delta f^g_{s_z/+} \equiv \Delta g$ for the helicity distribution functions, while $f_{q/p} \equiv q$ and $f_{g/p} \equiv g$ for the unpolarized distribution functions, for quarks and gluons respectively).

Our goal is to explore whether the mechanisms induced by the presence of partonic intrinsic transverse momenta, obtained in a general and fully non-collinear kinematics, could affect the above conclusions, based on the analysis of A_{LL} in the collinear configuration, i.e. taking into account only the terms proportional to $\Delta q(x)$ and $\Delta g(x)$. Could the "new" contributions shown in Eqs. (8)-(11) turn the longitudinal double asymmetry A_{LL} into a very small (or even slightly negative) quantity without the need to assume Δg to be zero or negative?

To answer this question, we study A_{LL} at RHIC, for the PHENIX kinematics: $\sqrt{s} = 200$ GeV and $|\eta| < 0.35$ (numerical calculations are performed at $\eta = 0$) and evaluate each separate contribution to A_{LL}, according to Eqs. (8)-(15). Since we have no knowledge of the parton densities $\Delta \hat{f}^q_{s_x/+}$ and $\Delta \hat{f}^g_{\mathcal{T}_2/+}$ we maximize them in order to see whether, in principle, they can have a significant effect on A_{LL}. We thus use for them the corresponding unpolarized parton densities and adjust the signs so that all contributions add up. Notice that we retain the full phase structure of each term, crucial for our purposes.

For the helicity distributions we have used the sets GRSV2000 [26] and LSS05 [27]. The unpolarized cross-section and the maximized contributions to the numerator of A_{LL} have been calculated using the GRV98 set [28] and the MRST01 set [29] respectively. For the fragmentation functions we have used the KKP set [30]. The transverse momentum dependence has been included by means of a factorized Gaussian smearing, for all the parton distribution and fragmentation functions

$$\hat{f}(x, k_\perp) = f(x)\, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}\,, \tag{17}$$

$$\hat{D}(z, k_{\perp C}) = D(z)\, \frac{e^{-k_{\perp C}^2/\langle k_{\perp C}^2 \rangle}}{\pi \langle k_{\perp C}^2 \rangle}\,, \tag{18}$$

with a constant and flavour independent parameter $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0$, assumed to be the same for all quark flavours and for gluons; we shall study the effect of changes in the value of k_0. Guided by our previous work, we compared the results obtained using three different values for k_0: $k_0 = 0.8$ GeV/c from studies on the unpolarized pp scattering cross-sections and single spin asymmetries [13], $k_0 = 0.5$ GeV/c from fitting the Cahn effect in SIDIS [31], and $k_0 = 0.01$ GeV/c to recover the collinear configuration. For the fragmentation functions, we take $\langle k_{\perp C}^2 \rangle = \langle k_\perp^2 \rangle$ everywhere, unless differently stated. Indeed we have checked that variations in $\langle k_{\perp C}^2 \rangle$ induce negligible changes in A_{LL}.

We have evaluated the contribution of the various terms to A_{LL} in the non-collinear approach. As already anticipated, we find that the contributions from gluon-gluon elementary scattering processes dominate at small values of p_T, while they become progressively less relevant for growing p_T's, as qg scattering contributions take over and become dominant for $p_T > 8$ GeV/c. The contribution from the collinear helicity distribution functions coming from channels involving only quarks and anti-quarks in the initial state *is* negative; however it is much smaller in magnitude than those corresponding to gluon-gluon and quark-gluon initial states, over the whole p_T range.

It turns out that the new "non collinear" soft contributions containing the PDF's $\Delta \hat{f}^q_{s_x/+}(x, k_\perp)$ and $\Delta \hat{f}^g_{\mathcal{T}_2/+}(x, k_\perp)$, even if maximized, are negligible. Therefore, we conclude that there is no way for the extra contributions induced by

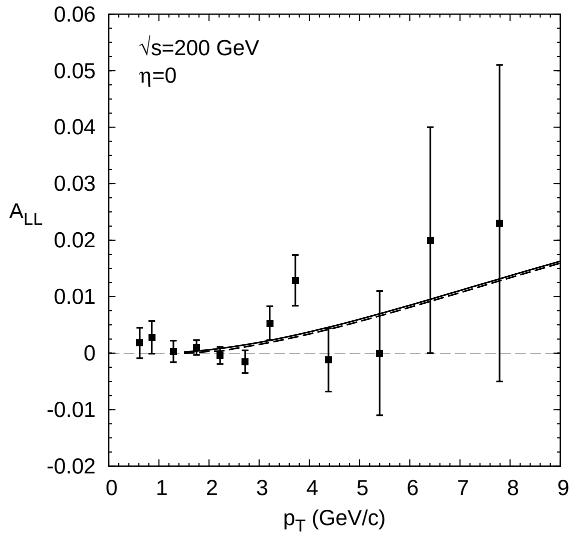

Figure 1: A_{LL} for the process $pp \to \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as function of p_T, calculated with different choices of $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0$ in the PDF/FFs, compared to PHENIX data, Run 5 [22]. The solid line corresponds to the choice $k_0 = 0.01$ GeV/c in both PDFs and FFs. The dashed line corresponds to $k_0 = 0.8$ GeV/c in PDF/FFs. The PDF sets are LSS05 [27] and MRST01 [29], the FF set is KKP [30] and the factorization scale is $Q = p_T$. Notice that the changes in A_{LL} induced by varying the value of $\langle k_\perp^2 \rangle$ are much smaller than those obtained by choosing different sets of distribution functions and/or factorization scales, see Fig. 2.

the presence of partonic intrinsic transverse momenta to alter the size of A_{LL}. This result does not depend on the choice of k_0.

As a further interesting issue we have studied the dependence of the usual leading contributions on $k_\perp$ and on the choice of the PDF set and of the factorization scale. In this study we have not taken into account the (negligible) contributions from the new soft functions.

As a first step, we have considered in more detail the dependence of A_{LL} on the mean transverse momentum of the PDFs. From Fig. 1 we can see that in general A_{LL} depends very little on the different choices of k_0; in particular, A_{LL} decreases when increasing the width of the gaussian, but compared to data this variation is quite negligible. This result was somehow expected, as the $k_\perp$ dependence is given by the same gaussian for all distribution and fragmentation functions and at mid rapidity the $\hat{M}$ amplitudes depend very mildly on $k_\perp$. In general, A_{LL} depends much more on the choice of the PDF set or of the scale. In Fig. 2 we show A_{LL} calculated in an almost collinear configuration, $\sqrt{\langle k_\perp^2 \rangle} = 0.01$ GeV/c, and for two choices of scale, $Q = p_T$ and $Q = p_T^*/2$, where p_T^* is the transverse momentum of the fragmenting parton in the partonic c.m. frame. Using the LSS05/MRST01 PDFs, the solid line corresponds to $Q = p_T^*/2$ and the dashed-dotted line to $Q = p_T$. Using the GRSV2000/GRV98 PDFs, the dashed line corresponds to $Q = p_T^*/2$, and the dotted line to $Q = p_T$. As it can be seen, the variations induced by different choices of PDF set and scale are quite large, larger than those produced by changes in the k_0 value; nevertheless, all these curves are compatible with experimental data.

Furthermore, we have studied the dependence of A_{LL} on the c.m. energy. The left panel of Fig. 3 shows A_{LL} at $\sqrt{s} = 62.4$ GeV and $\sqrt{s} = 200$ GeV at $\eta = 0$ calculated with $\sqrt{\langle k_\perp^2 \rangle} \equiv k_0 = 0.5$ GeV/c. It is seen that A_{LL} varies considerably at fixed $x_T = 2\,p_T/\sqrt{s}$, but remains very small. Notice that preliminary experimental data obtained by the PHENIX Collaboration during a short run at $\sqrt{s} = 62.4$ GeV have been recently presented [23]: they confirm that A_{LL} actually becomes smaller at lower $\sqrt{s}$, but, with the present large errors, do not offer a realistic possibility to distinguish among different possible scenarios for Δg.

Even in the range of forward rapidities, A_{LL} remains very small. In Fig. 3, right panel, we show A_{LL} at three different rapidity values, as a function of x_F. Clearly, A_{LL} quickly decreases when η becomes larger than one, making it impossible to study any variations induced by different choices of PDF/FFs or by changes in k_0.

To complete the picture we have examined a more general case in which the dependence on $k_\perp$ of the helicity distribution functions, Δq and Δg, are parametrized in terms of two different gaussians, namely

$$\Delta \hat{f}_{s_z/+}(x, k_\perp) = \hat{f}_{+/+}(x, k_\perp) - \hat{f}_{-/+}(x, k_\perp) = f_{+/+}(x)\, \frac{\exp(-k_\perp^2/\langle k_+^2 \rangle)}{\pi \langle k_+^2 \rangle} - f_{-/+}(x)\, \frac{\exp(-k_\perp^2/\langle k_-^2 \rangle)}{\pi \langle k_-^2 \rangle}\,, \tag{19}$$

to allow for a non-trivial interplay between $\hat{f}_{+/+}$ and $\hat{f}_{-/+}$ ($\hat{f}_{\pm/+}$ is the number density of partons with $\pm$ helicity

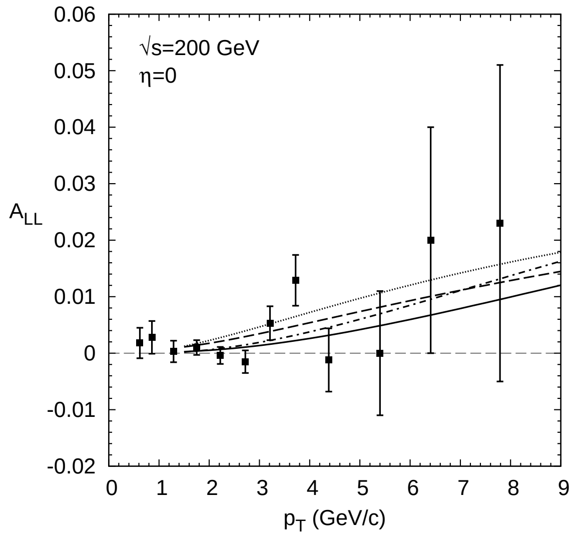

Figure 2: A_{LL} for the process $pp \to \pi^0 X$ at $\sqrt{s} = 200$ GeV and $\eta = 0$, plotted as function of p_T, calculated with different PDF sets and factorization scales. The solid line corresponds to the choice of PDFs LSS05/MRST01, while the factorization scale is $Q = p_T^*/2$, where p_T^* is the transverse momentum of the fragmenting parton in the partonic c.m. frame. The dash-dotted line corresponds to LSS05/MRST01 and $Q = p_T$. The dashed line shows A_{LL} as obtained using the PDF sets GRSV2000/GRV98 and with $Q = p_T^*/2$. Finally the dotted line corresponds to the choice GRSV2000/GRV98 and $Q = p_T$. The FF set is KKP. $\sqrt{\langle k_\perp^2 \rangle} = 0.01$ GeV/c for both PDF/FF. The experimental data are from the PHENIX collaboration at RHIC, Run 5 [22].

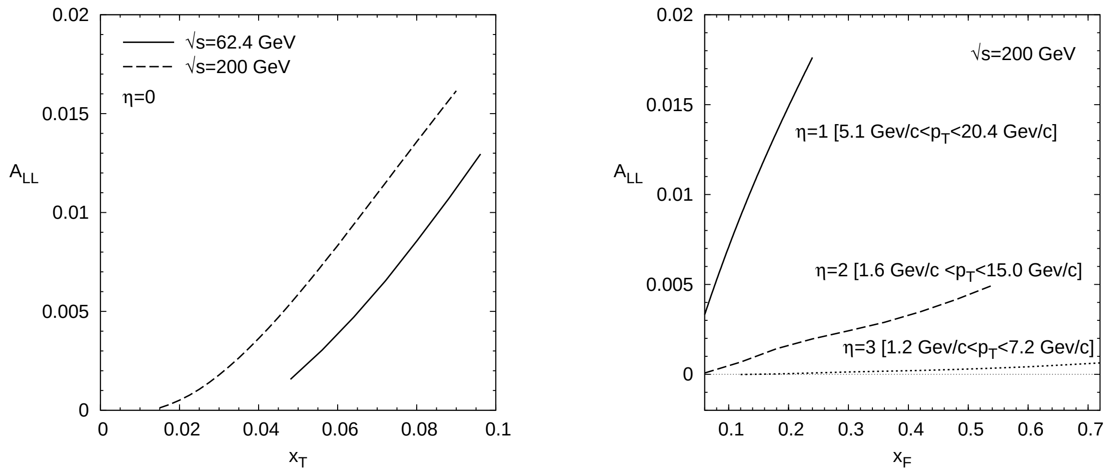

Figure 3: On the left panel, A_{LL} is shown as a function of x_T at rapidity $\eta = 0$, factorization scale $Q = p_T$ and two different c.m. energies: the solid line corresponds to $\sqrt{s} = 62.4$ GeV while the dashed line to $\sqrt{s} = 200$ GeV. On the right panel, A_{LL} is plotted as function of x_F at $\sqrt{s} = 200$ GeV and different values of rapidity, $\eta = 1, 2, 3$; here the factorization scale is $Q = p_T^*/2$. The PDF sets are LSS05 and MRST01, the FF set is KKP and $k_0 = 0.5$ GeV/c for both PDF/FF.

inside a positive helicity parent proton). Notice that, for simplicity, we have assumed no flavour dependence: the same $\langle k_+^2 \rangle$ and $\langle k_-^2 \rangle$ parameters are used for any quark flavour and for gluons, in hadrons A and B. Once again we have found that A_{LL} is very little sensitive to different choices for the width parameters $\langle k_+^2 \rangle$ and $\langle k_-^2 \rangle$ at mid rapidity. The consequences of using different gaussians for the two helicity components of the PDFs can be seen in Fig. 4 where A_{LL} is plotted as a function of x_F at $\sqrt{s} = 200$ GeV and three different rapidities, $\eta = 1$ (left panel), $\eta = 2$ (central panel) and $\eta = 3$ (right panel). Here we adopt $\sqrt{\langle k_{\perp C}^2 \rangle} = 0.5$ GeV/c for the FFs, while the four lines represent four different choices for the parameters $\langle k_+^2 \rangle$ and $\langle k_-^2 \rangle$. It is immediately evident that variations in these parameters induce very little change in A_{LL}, except at larger rapidity, where, however, A_{LL} is so small that it is probably beyond experimental reach. Notice the scale of the pictures, many times smaller than that used in Figs. 1 and 2. It is also interesting to point out that the curves that correspond to the choices $\langle k_+^2 \rangle = \langle k_-^2 \rangle$ (which leads us back to the cases

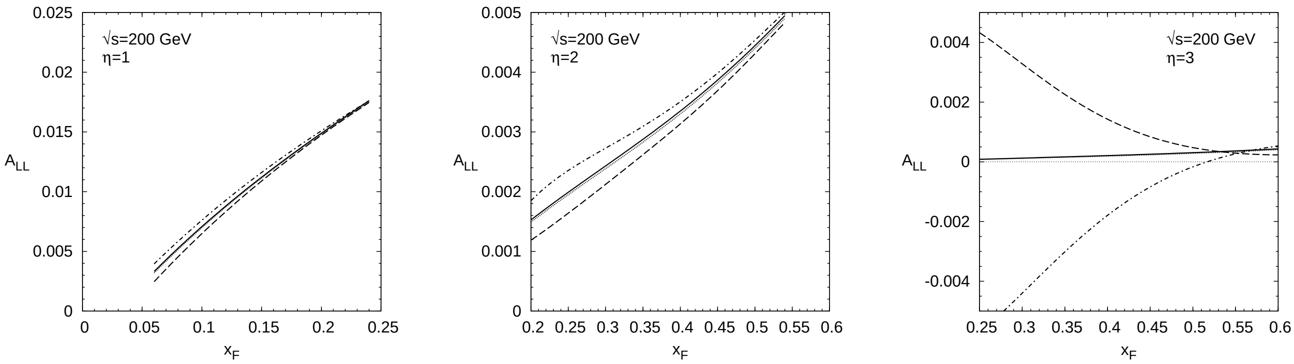

Figure 4: The longitudinal double spin asymmetry A_{LL} plotted as function of x_F at $\sqrt{s} = 200$ GeV and rapidities $\eta = 1$ (left panel), $\eta = 2$ (central panel) and $\eta = 3$ (right panel). The factorization scale is $Q = p_T^*/2$, where p_T^* is the transverse momentum of the fragmenting parton in the partonic c.m. frame. The four lines represent four different choices for the parameters $\langle k_+^2 \rangle$ and $\langle k_-^2 \rangle$: the solid line refers to $\langle k_+^2 \rangle = \langle k_-^2 \rangle = 0.04$ (GeV/c)^2, the dashed line to $\langle k_+^2 \rangle = 0.36$ (GeV/c)^2, $\langle k_-^2 \rangle = 0.04$ (GeV/c)^2, the dash-dotted line to $\langle k_+^2 \rangle = 0.04$ (GeV/c)^2, $\langle k_-^2 \rangle = 0.36$ (GeV/c)^2, and the dotted line refers to $\langle k_+^2 \rangle = \langle k_-^2 \rangle = 0.36$ (GeV/c)^2. The PDF sets are LSS05 and MRST01, the FF set is KKP. Here we adopt $\langle k_{\perp C}^2 \rangle = 0.25$ (GeV/c)^2 for the FFs. Notice that the scale here is many times smaller than in Figs. 1 and 2.

previously considered) are basically indistinguishable, independently of the value assumed for the two widths.

V. CONCLUSIONS

We have examined, at leading order in perturbative QCD, the effect on the longitudinal double spin asymmetry A_{LL} of allowing the partons to have non-zero intrinsic transverse momentum, and of including in A_{LL} the contributions arising from the new soft functions that play a crucial role in transverse spin asymmetries. The study was carried out in the hope that such effects might negate the conclusion that the very small measured values of A_{LL} automatically imply that the polarized gluon density is very small. Our study indicates that the contribution from these effects is essentially negligible and we are forced, at the present stage, to accept the conclusion that the polarized gluon density is much too small to explain the "spin crisis in the parton model" [32]. In fact, even in a fully non-collinear partonic kinematics, the variations induced by changes in the parameter $\langle k_\perp^2 \rangle = k_0$ are very small, much smaller than those generated by different choices of PDF sets and of the factorization scale. Finally, we have studied the behaviour of the longitudinal double spin asymmetry A_{LL} at different c.m. energies and at different rapidities, in a model in which two different widths were assumed for the parallel and antiparallel-spin parton densities, $\hat{f}_{+/+}$ and $\hat{f}_{-/+}$: it turns out that A_{LL} remains small and positive in all cases, except at large rapidity where its magnitude is probably too small to be measurable.

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