

Comments on the Blümlein-Böttcher determination of higher twist corrections to the nucleon spin structure function g_1

Elliot Leader

*Imperial College London
Prince Consort Road, London SW7 2BW, England*

Aleksander V. Sidorov

*Bogoliubov Theoretical Laboratory
Joint Institute for Nuclear Research, 141980 Dubna, Russia*

Dimitar B. Stamenov

*Institute for Nuclear Research and Nuclear Energy
Bulgarian Academy of Sciences
Blvd. Tsarigradsko Chaussee 72, Sofia 1784, Bulgaria*

Abstract

In a recent analysis of the world data on polarized DIS, Blümlein and Böttcher conclude that there is no evidence for higher twist contributions, in contrast to the claim of the LSS group, who find evidence for significant higher twist effects. We explain the origin of the apparent contradiction between these results.

PACS numbers: 13.60.Hb, 12.38.-t, 14.20.Dh

1 Introduction

Because of the limited range of Q^2 available in the world data sample for polarized DIS, and because of the great accuracy of some of the data at relatively low values of Q^2 , it is important to be able to include the latter data in QCD analyses aimed at extracting information on the polarized parton densities. To do this consistently the theoretical formulae must be extended beyond leading twist (LT) to allow for higher twist (HT) contributions. This has been done for some years by Leader, Sidorov and Stamenov (LSS) [1], who have shown that such contributions are important, especially in fitting the very accurate CLAS data [2]. In contrast, Blümlein and Böttcher (BB) [3] claim that HT effects are completely negligible. In this note we explain that the disagreement is probably only apparent, and results from attempting to compare two incommensurate quantities.

2 Structure of higher twist terms

On the basis of the Operator Product Expansion (OPE), LSS have utilized an expression for the experimental spin dependent structure function $g_1(x, Q^2)_{exp}$ of the form

$$\begin{aligned} g_1(x, Q^2)_{exp} &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \\ &= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} + \mathcal{O}(\Lambda^4/Q^4), \end{aligned} \quad (1)$$

where the powers in $1/Q^2$ corrections to g_1 have been split into the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT). In the LSS analysis the logarithmic Q^2 dependence of h , which is unknown in QCD, is neglected. Compared to the principal $1/Q^2$ dependence it is expected to be small and the accuracy of the present data does not allow its determination. Therefore, the extracted from the data values of $h(x)$ correspond to the mean Q^2 for each x -bin. The most recent results for $h(x)$ for proton and neutron targets [4] are shown in Fig.1.

BB write the expression for $g_1(x, Q^2)_{exp}$ in a different form, namely

$$g_1(x, Q^2)_{exp} = g_1(x, Q^2)_{LT} \left[1 + \frac{C(x)}{Q^2} \right], \quad (2)$$

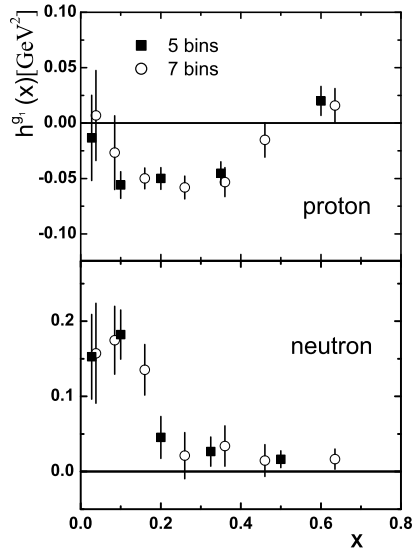


Figure 1. Higher twist corrections extracted from the world polarized DIS data (corresponding to 5 and 7 x bins).

where the assumption

$$g_1(x, Q^2)_{HT} = g_1(x, Q^2)_{LT} \frac{C(x)}{Q^2} \quad (3)$$

is used for the twist-4 contribution, and any Q^2 dependence in $C(x)$ is neglected. (It is not clear whether the TMC are accounted for in $g_1(x, Q^2)_{LT}$.)

BB find no evidence for HT i.e. their $C(x)$ for protons and neutrons is compatible with zero. (There is a worrying issue concerning their derivation of the neutron value, which we shall comment on later.)

In trying to understand the apparent discrepancy with LSS, BB write: "This result is in disagreement to Ref. [21] (i.e. LSS [5]). Note that in the latter analysis a partonic description of $F_1(x, Q^2)$ down to low values of Q^2 is used, while we refer to the measured function."

This statement originates in the fact that what is actually measured is effectively $g_1(x, Q^2)_{exp}/F_1(x, Q^2)_{exp}$, so that $g_1(x, Q^2)_{exp}$ must be extracted from the data by multiplying by $F_1(x, Q^2)_{exp}$.

However, the comment about Ref. [21] (Ref. [5] in this paper) is totally incorrect. Indeed in [5] LSS write:

”According to this method, the g_1/F_1 and A_1 data have been fitted using the experimental data for the unpolarized structure function $F_1(x, Q^2)$

$$\left[\frac{g_1(x, Q^2)}{F_1(x, Q^2)} \right]_{exp} = \frac{g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + h(x)/Q^2}{F_1(x, Q^2)_{exp}}. \quad (4)$$

As usual, F_1 is replaced by its expression in terms of F_2 and R , and phenomenological parametrizations of the experimental data for $F_2(x, Q^2)$ [8] and the ratio $R(x, Q^2)$ of the longitudinal to transverse cross-sections [9] are used.”

Hence the apparent discrepancy between BB and LSS is certainly not due to any difference in the handling of $F_1(x, Q^2)$.

3 Why the discrepancy is only apparent

On equating Eqs. (1) and (2) we have

$$C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}}. \quad (5)$$

Thus, if it is legitimate to neglect the scale dependence in $h(x)$ then $C(x)$ must vary with Q^2 , contradicting the use of $C(x)$ as Q^2 -independent. If, on the other hand, it is legitimate to neglect the Q^2 dependence in $C(x)$, then $h(x)$ must vary with Q^2 . We thus see that the two approaches are incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect and the fact that their results disagree is inevitable and requires no further explanation. However, since the LSS formulation is closer in structure to the operator product expansion, we believe it is more likely to be the correct way to implement HT corrections. Moreover, the LSS results on HT are in good agreement with those obtained from the study of the *first* moments of the spin structure functions $g_1^{(p,n)}(x, Q^2)$, and in particular, of the non-singlet structure function $g_1^{(p-n)}$ (see Ref. [6]).

One further point remains. BB utilize Eq. (2) for proton and deuteron data and then extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d(x) - C_p(x) \quad (6)$$

where $\omega_D = 0.05 \pm 0.01$. From Eq. (2) one sees that this is incorrect. The correct relation should be

$$C_n(x) = \frac{1}{g_{1n}(x, Q^2)_{LT}} \left[\frac{2}{1 - 1.5\omega_D} g_{1d}(x, Q^2)_{LT} C_d(x) - g_{1p}(x, Q^2)_{LT} C_p(x) \right] \quad (7)$$

Thus, even if it is correct to take $C_p(x)$ and $C_d(x)$ independent of Q^2 , C_n will inevitably inherit some dependence on Q^2 . Note also that the neutron spin structure function $g_{1n}(x, Q^2)_{LT}$ passes through zero as a function of x and it is therefore dangerous to use the above equation to extract the HT correction C_n .

4 Conclusions

We have shown that the LSS and BB methods of extracting HT corrections in polarized DIS are incompatible and that it thus makes no sense to compare their results - they are incommensurate. We believe that the LSS approach, because it is closely related to the operator product expansion, is more likely to be the correct one.

Acknowledgments

This research was supported by the JINR-Bulgaria Collaborative Grant, by the RFBR Grants (No 09-02-01149, 10-02-01259) and by the Bulgarian National Science Foundation under Contract 288/2008. One of the authors (E.L) is grateful to the participants at the Brookhaven Summer Program on Nucleon Spin Physics for stimulating comments, and to the organizers for support.

References

- [1] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Rev. D **67**, 074017 (2003).
- [2] K.V. Dharmawardane *et al.* (CLAS Collaboration), Phys. Lett. B **641**, 11 (2006)
- [3] J. Blümlein and H. Böttcher, arXiv:1005.3113 [hep-ph].

- [4] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Rev. D **75**, 074027 (2007).
- [5] E. Leader, A.V. Sidorov, and D.B. Stamenov, Phys. Rev. D **80**, 054026 (2009).
- [6] A. Deur et al., Phys. Rev. D **78**, 032001, 2008.; R. Pasechnik, D. V. Shirkov, and O. V. Teryaev, Phys. Rev. D **78**, 071902, 2008.